Bottom-up Markup Fluctuations[*]

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Abstract

We study markup cyclicality in a granular macroeconomic model with oligopolistic competition. We characterize the comovement of firm, sectoral, and economy-wide markups with sectoral and aggregate output following firm-level shocks. We then quantify the model's ability to reproduce salient features of the cyclical properties of measured markups in French administrative firm-level data, from the bottom (firm) level to the aggregate level. Our model helps rationalize various, seemingly conflicting, measures of markup cyclicality in the French data.

Keywords: Markup Cyclicality; Oligopolistic Competition; Firm Dynamics; Granularity; Aggregate Fluctuations

Introduction

A long tradition in the business-cycle literature evaluates models on their ability to account for salient moments of business-cycle data, such as the relative volatility and correlation with GDP of key macroeconomic aggregates. Although there exists a broad consensus on moments concerning, for example, the behavior of consumption, investment, or unemployment over the business cycle, disagreement lingers, both in terms of theory and measurement, over the cyclical behaviour of markups (see, e.g., Bils et al., 2018 and Nekarda and Ramey, 2013, 2020).[1]

In this paper, we re-examine this question, studying the cyclical properties of markups at the firm, sector, and aggregate level, both theoretically and empirically, based on French administrative data. We consider a model of oligopolistic competition in which granular firm-level shocks result not only in sector and economy-wide output changes, as in Gabaix (2011), but also in markup dynamics. We characterize the model's implications for comovement between output and markups, that is "markup cylicality", at various levels of disaggregation from the bottom (firm) level up to the sector and aggregate levels, and show how this comovement is mediated by market structure within and across sectors. We then assess the quantitative ability of our granular oligopolistic setting to reproduce salient measures of the cyclical properties of markups in the French data at the firm, sector, and aggregate levels.

To model in a tractable way the determination and aggregation of markups in an economy featuring a large but finite number of sectors with a discrete number of firms, we use a nested CES demand structure studied in Atkeson and Burstein (2008).[2] Firms compete under flexible prices, setting markups that are increasing in within-sector sales shares.[3] Firm-level shocks follow a random growth process that generates empirically plausible firm dynamics, firm-size distributions, and granular sectoral and aggregate fluctuations (Carvalho and Grassi 2019). The model yields predictions for the joint behavior of within-sector market shares, markups, and output following exogenous changes in firm-level shifters. Furthermore, the model's conve-

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[1]Other studies contributing to the active debate on the sign and magnitude of markup cyclicality include Bils (1987), Hall (1988), Anderson, Rebelo and Wong (2018), and Stroebel and Vavra (2019). Additionally, a growing literature provides measures of lower-frequency trends in markups, such as De Loecker, Eeckhout and Unger (2020) and De Loecker and Eeckhout (2018).

[2]A similar framework has been used in a number of macro applications to quantify the welfare costs of market power (Edmond, Midrigan and Xu, 2018 and Berger, Herkenhoff and Mongey, 2019), trends in market power (De Loecker, Eeckhout and Mongey, 2018), optimal product market policy (Boar and Midrigan, 2019), managerial compensation (Bao, De Loecker and Eeckhout, 2022), wage inequality (Deb, Eeckhout, Patel and Warren, 2022), pro-competitive gains from trade (Edmond, Midrigan and Xu, 2015), exchange-rate pass-through (Amiti, Itskhoki and Konings, 2019), and granularity in trade (Gaubert and Itskhoki, 2018). Other prominent work featuring fluctuations in market power in macroeconomic models include Gali (1994), Kimball (1995), Jaimovich and Floetotto (2008), and Bilbiie, Ghironi and Melitz (2012).

[3]Much of the literature on markup cyclicality is motivated by the implications of models with nominal rigidities (e.g., Rotemberg and Woodford (1999) for a comprehensive early survey), which depend on the nature of nominal rigidities (prices vs. wages) and on the source of aggregate shocks (e.g., monetary vs. productivity). By contrast, we examine how far a model with flexible prices and granular, firm-level shocks can go in accounting for observed patterns of markup cyclicality at different levels of disaggregation. See Mongey (2017) and Wang and Werning (2020) for recent analyses of money non-neutrality in an oligopolistic model like ours with price rigidities.

nient equilibrium aggregation yields simple sectoral and aggregate counterparts to many of these firm-level objects.

Our first theoretical contribution is to provide simple analytic expressions showing how the sign of markup cyclicality depends on the level of aggregation, the market structure within and across sectors, and the set of shocked firms.[4] We show sectoral output and markups comove positively in response to shocks to large firms in the sector, whereas they comove negatively in response to shocks to small firms. In turn, the effect of such shocks on the aggregate markup depends on the distribution of sector-level markups and sectoral expenditure shares. Under the additional assumption that shocks are uncorrelated across firms (such that large firms drive the cycle in each sector), we provide sufficient conditions for a positive asymptotic correlation between markups and output at the sectoral and aggregate levels.

Second, we compare, theoretically, the implications of our model to an alternative specification in which firm-level markups are heterogeneous but constant in response to shocks (i.e., complete pass-through) so that sectoral and aggregate markups only change due to between-firm reallocation and not within-firm markup changes. We show that, although within-firm markup changes account for half of sectoral markup fluctuations in the variable markup model, changes in sectoral and aggregate markups can be larger or smaller than in the constant markup model, depending on parameter values, because the extent of between-firm reallocation falls with incomplete pass-through. Additionally, we provide analytical formulas for sectoral and aggregate output volatility, which generalize those in Gabaix (2011) to an oligopolistic setting with variable markups, and show how the introduction of variable markups dampens granular aggregate volatility, by acting in a similar way to a decline in the Herfindahl index. Intuitively, when pass-through rates are lower for larger firms, the weight of large firm shocks in the price index is effectively reduced relative to constant markup models.

Our quantitative analysis is based on French administrative firm-level data over a 26-year period (1994-2019) covering approximately 400,000 firms per year. We use these data to compute empirical distributions of firm market shares, sectoral output and concentration, and aggregate output over our annual sample. We use balance-sheet, price and quantity information to obtain a measure of firm-level markups – following the methodologies in De Loecker and Warzynski (2012), De Loecker et al. (2016) and De Ridder et al. (2022) – which we then aggregate at the sectoral and economy-wide levels.[5,6] We employ a rich set of empirical moments at

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[4]Grassi (2018) studies the role of input-output linkages and endogenous markups in shaping comovement of sector-level variables, providing an analytical characterization of the impact of microeconomic shocks on aggregate output using an approximation with respect to the deep parameters of the model. Our analytic results make use of a different approximation with respect to firm-level idiosyncratic shocks, similar to the one used in, for example, Gopinath et al. (2010), Burstein and Gopinath (2014), and Amiti et al. (2019) in the context of exchange rate shocks.

[5]By using information on firm-level quantities, our measures of markups are not prone to the biases identified in Bond et al. (2020) that result from using revenues rather than quantities. See De Ridder et al. (2022) for a detailed discussion.

[6]Given the level of aggregation in our baseline dataset, we do not measure markups at the level of geographic regions, as in Anderson et al. (2018), or products, as in De Loecker et al. (2016). For consideration of pricing with

the firm, sector, and aggregate levels both to calibrate our model and to assess its quantitative implications for (non-targeted) markup cyclicality.

We start by analyzing a basic mechanism in our oligopolistic setting: within a narrowly defined sector, a firms' market power is increasing in its market share. This relationship implies a positive correlation between firm-level markup and firm-level market share, both in the cross section and over time. Moreover, aggregating firm-level outcomes implies the same correlation between sectoral markups and sectoral concentration. We find support for these predicted correlations in the French data, both at the firm and at the sector levels, in the cross section, and over time.[7] Moreover, changes in firm-level markups account for a sizable portion of changes in sectoral markups both in our model (50%) and in the data (53% in the median sector).[8]

Second, we examine in the model and data three measures of markup cyclicality in the literature. We first consider a notion of firm-level markup cyclicality proposed by Hong (2017). In particular, we ask whether firm-level markups covary systematically with respect to sector-level output. We find this reduced-form relation is "counter-cyclical" for the average firm in the French data, but switches sign for large firms. Consistently with the model mechanics discussed above, our quantitative model is able to reproduce these findings. Relatedly, we additionally find that, in the data, large firms are "pro-cyclical" in that their market share increases during sectoral expansions, which is a key implication of our granular model.We then proceed to evaluate notions of sector-level markup cyclicality. Following Nekarda and Ramey (2013) (the working paper version of Nekarda and Ramey, 2020), we ask whether sector markups comove with sector output over the business cycle. Like Nekarda and Ramey (2013) for the US, we find evidence for a positive systematic comovement between the two measures, or "pro-cyclicality", in the French data. The model simulations also reproduce this fact, as anticipated in our theoretical discussion. Also consistently with the model mechanism, we find that in the data, sectoral expansions tend to be associated with an increase in sectoral concentration Finally, we follow the work of Bils et al. (2018), who investigate yet another notion of cyclicality: the extent to which sector level markups comove with aggregate output. According to this measure, we find weak (statistically insignificant) counter-cyclical point estimates. Bils et al. (2018) document a negative correlation for the US. Simulations of our model also imply that this alternative comovement measure is not statistically different from zero. Overall, in the data, we obtain seemingly conflicting measures of markup cyclicality across different layers of aggregation despite using a single dataset and measure of firm-level markups. This suggests that by exploiting different reduced-form measures of markup cyclicality, two researchers may arrive at opposing conclusions even within a single dataset. Nevertheless, our model can reproduce

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multi-product firms in a similar framework to ours, see Hottman et al. (2016).

[7]Relatedly, Brooks et al. (2016) and Kikkawa et al. (2019) provide evidence of a positive relation between market shares and markups in the time series using firm-level data from China and Belgium, respectively.

[8]This is consistent with Figure 3 in Baqaee and Farhi (2019), showing that within-firm changes in markups in the US are quantitatively important in accounting for high frequency movements in aggregate markup.

qualitatively, and sometimes quantitatively, these different reduce-form notions measures of markup cyclicality.

Finally, we examine the model's implications for aggregate markup and output fluctuations. In our baseline calibration we abstract from aggregate shocks that leave the firm-size distribution unchanged because, in our model, they do not affect markups. Our model with granular firm-level shocks generates roughly 25% (on average, across 25-year simulated samples) of the volatility of aggregate output in the French data. The ratio of markup volatility relative to output volatility is roughly 0.7 in the data and 0.4 in the model, abstracting from other aggregate shocks. Note that much of the work on the granular origin of business cycles abstracts from these movements in desired markups that can partly offset the impact of own firm-level shocks or magnify the impact from shocks to competing firms.[9,10] Turning to aggregate markup cyclicality, our model implies a counterfactually large and positive point estimate for the correlation between aggregate output and markups relative to the data. However, there is large variation in point estimates across small 25-year simulated samples. This is because, as our analytic expressions show, the extent of markup cyclicality depends on the set of shocked firms, which can vary substantially across small samples. Moreover, superimposing aggregate productivity shocks to account for the overall aggregate volatility reduces this correlation significantly. Finally, we show the magnitude and cyclicality of aggregate markups in our model is not too different when we counterfactually fix markups at their initial, heterogeneous equilibrium level. Of course, rather than exogenously fixing markups, our model provides a unified theory of both markup (level) heterogeneity across firms and endogenous markup changes.

The paper is organized as follows. In section 1, we present our granular oligopolistic setup and describe the equilibrium from the bottom (firm) level to the aggregate level. In section 2, we characterize analytically various measures of markup cyclicality at various aggregation levels. In section 3, we discuss our French administrative firm data, the markup-estimation strategy and model calibration. In section 4, we compare a host of markup-related moments in the data and in model-generated data. Section 5 concludes. In the Appendix, we more fully discuss markup estimation, provide additional results and proofs, and present robustness checks.

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[9]Gaubert and Itskhoki (2018) study the granular origins of a country's comparative advantage in an oligopolistic framework that is similar to ours. For work on the granular origins of business-cycle fluctuations – but featuring either perfect competition or constant markups – see Carvalho and Gabaix (2013) on the evolution of business-cycle volatility over time and across countries and di Giovanni and Levchenko (2012) or di Giovanni et al. (2018) for granular settings linking trade, aggregate volatility, and cross-country comovement. di Giovanni et al. (2014) provide an empirical benchmark for the role of granularity in aggregate fluctuations. Our emphasis on the micro origins of aggregate fluctuations is also related to the literature on production networks. See Acemoglu et al. (2012) for an initial benchmark, and Grassi (2018) for an analysis of how market power distorts the propagation of shocks along input linkages. Finally, Pasten et al. (2020) examine aggregate granular fluctuations in a multi-sector model allowing for changes in markups due to nominal price rigidities.

[10]Baqaee and Farhi (2019) provide a very general characterization of the impact of microeconomic shocks on aggregate productivity and output in a large class of models in which productivities and wedges (e.g., markups) are exogenous primitives. Baqaee and Farhi (2020) study the role of variable markups in shaping the aggregate implications of changes in market size.

1   Model

Our model consists of a representative household that supplies labor and consumes a fixed set of goods produced by a discrete number of flexible-price firms that compete oligopolistically and that the representative household owns. In this section we describe the model and characterize the equilibrium, first within a sector and then at the aggregate level.

1.1   Preferences and technologies

Households have preferences at time t over consumption of a final composite good, Y_t, and labor, L_t, represented by the utility function

$$U\left(Y_t, L_t\right) = \frac{1}{1-\eta} Y_t^{1-\eta} - \frac{f_0}{1+f^{-1}} L_t^{1+f^{-1}},$$

where $\eta \leq 1$ and $f \geq 0$ are, respectively, the constant relative risk aversion and the Frisch elasticity of labor supply. Households choose consumption and labor to maximize utility subject to the constraint that consumption expenditures must not exceed the sum of wage payments and aggregate profits.

The final good aggregates output of N sectors according to a constant-elasticity-of-substitution (CES) aggregator:

$$Y_t = \left[\sum_{k=1}^{N} A_k^{\frac{1}{\sigma}} Y_{kt}^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where Y_{kt} denotes sector k output, A_k is a demand shifter for sector k (which we assume is constant over time, and we set it in our calibration to target the average size of each sector), and $\sigma \geq 1$ is the elasticity of substitution across sectors.

As in Atkeson and Burstein (2008), each sector k is itself a CES aggregator of the output of N_k individual firms given by

$$Y_{kt} = \left[\sum_{i=1}^{N_k} A_{kit}^{\frac{1}{\varepsilon}} Y_{kit}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where Y_{kit} denotes the output of firm i in sector k, A_{kit} is a firm-quality shifter, and ε is the elasticity of substitution between the output of firms in sector k.[11] We assume $\sigma \leq \varepsilon$, so that goods are more substitutable within sectors than across sectors. With a finite number of sectors and a discrete number of firms per sector, firm-level shocks can generate aggregate fluctuations as in Gabaix (2011). By contrast, with a continuum of sectors, as in Atkeson and Burstein (2008),

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[11]The model's implications for markups, market shares, and concentration measures are unchanged if A_{kit} is a taste shock. However, measures of aggregate output calculated using chain-weighted deflators are path-dependent in the presence of taste shocks (i.e. growth between t and t' depends on the sequence of shocks between t and t'); see e.g. Baqaee and Burstein (2021). For this reason, we abstract from taste shocks. Given the challenges of measuring quality changes at high frequencies, in the quantification we only consider firm-level productivity shocks.

firm-level shocks would not generate aggregate fluctuations.

Firm i in sector k produces output according to the constant-returns-to-scale technology:

$$Y_{kit} = Z_{kit}L_{kit}, \tag{1}$$

where Z_{kit} denotes the productivity of firm i in sector k and L_{kit} is a variable input, employment, that is perfectly mobile across firms.[12] Labor market clearing requires that the sum of employment across all firms equals aggregate labor, L_t.[13]

We introduce assumptions about the stochastic process of firm-level shocks A_{kit} and Z_{kit} in the analytic section 2 for our asymptotic results and in the quantitative section 3 when describing our model calibration.

1.2 Market structure and sector equilibrium

We now describe the equilibrium in a sector. Firm i in sector k setting a non-quality adjusted price P_{kit} faces demand $Y_{kit} = A_k A_{kit} (P_{kit})^{-\varepsilon} (P_{kt})^{\varepsilon - \sigma} P_t^{\sigma} Y_t$, where the sector k price is

$$P_{kt} = \left[\sum_{i=1}^{N_k} A_{kit} P_{kit}^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{2}$$

and the aggregate price is

$$P_t = \left[\sum_{k=1}^{N} A_k P_{kt}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

The markup for firm i in sector k, which we characterize below, is defined as the ratio of price to marginal cost,

$$\mu_{kit} \equiv \frac{Z_{kit}P_{kit}}{W_t}, \tag{3}$$

where W_t is the price of the variable input (i.e., the wage). This markup determines how the firm's revenues are split into labor payments and profits, such that

$$L_{kit}W_t = \mu_{kit}^{-1}P_{kit}Y_{kit}, \qquad \text{and} \qquad \Pi_{kit} = \left(1 - \mu_{kit}^{-1}\right)P_{kit}Y_{kit}.$$

The market share of firm i in sector k, $s_{kit} \equiv \frac{P_{kit}Y_{kit}}{P_{kt}Y_{kt}}$, can be expressed in terms of markups

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[12]In appendix A.7, we provide analytic results allowing for decreasing returns to scale at the firm level.

[13]Our results in section 2 on firm-level and sectoral-level outcomes are unchanged if the variable input, L_{kit}, is a composite of multiple inputs (e.g., labor, intermediate goods, and capital) that is common across firms in the sector. The specific assumptions on the composition of this variable input matter only for the aggregate response of the economy to given firm-level shocks. When estimating markups in Section 3, we assume the input L_{kit} is a translog combination of labor, capital, materials, and services inputs with parameters that vary by sector. We then compare measures of cyclicality of estimated markups in the data with measures of cyclicality implied by our model in section 4.

and *firm shifters*, which are defined as a composite of quality and productivity shifters, $V_{kit} \equiv A_{kit} Z_{kit}^{\varepsilon-1}$. Specifically,

$$s_{kit} = \frac{V_{kit}\mu_{kit}^{1-\varepsilon}}{\sum_{i'=1}^{N_k} V_{ki't}\mu_{ki't}^{1-\varepsilon}}. \tag{4}$$

One can consider two alternative market structures. Firms maximize profits by choosing price, taking other firms' prices as given (Bertrand competition), or by choosing quantity, taking other firms' quantities as given (Cournot competition). In both cases, firms take into account that they are non-atomistic in their sector, and hence their choices affect sectoral output and prices. We assume, however, that individual firms behave as if the sector they produce in is atomistic in the aggregate economy (as in the case of a continuum of sectors).[14]

Under these assumptions, equilibrium markups and market shares in each sector k solve the non-linear system of equations given by (4) and

$$\mu_{kit} = \begin{cases} \frac{\varepsilon}{\varepsilon-1}\left[1 - \left(\frac{\varepsilon/\sigma-1}{\varepsilon-1}\right)s_{kit}\right]^{-1} & \text{under Cournot,} \\[2em] \frac{\varepsilon}{\varepsilon-1}\left[\frac{1-\left(\frac{\varepsilon-\sigma}{\varepsilon}\right)s_{kit}}{1-\left(\frac{\varepsilon-\sigma}{\varepsilon-1}\right)s_{kit}}\right] & \text{under Bertrand.} \end{cases} \tag{5}$$

Under both formulations, since $\varepsilon > \sigma$, markups are increasing in market shares,[15] with $\lim_{s_{kit}\to 0}\mu_{kit} = \frac{\varepsilon}{\varepsilon-1}$ and $\lim_{s_{kit}\to 1}\mu_{kit} = \frac{\sigma}{\sigma-1}$. If $\varepsilon = \sigma$, markups are common across firms and constant over time as in the standard monopolistically competitive model. In our analytic and quantitative results, we focus on the case of Cournot competition because it generates more markup variation than Bertrand and is thus better able to match estimates of incomplete pass-through and markup-size relationship. In Appendix A, we provide analytic results under Bertrand.

Two remarks are in order about firm shifters. First, firm-level market shares and markups in sector k depend only on relative firm shifters across firms within this sector. This result implies that market shares and markups in sector k do not vary in response to proportional changes in shifters to all firms in sector k (including sectoral demand shifters A_k), shocks in other sectors, or changes in the aggregate wage. It follows that aggregate shocks to firms in all sectors generate fluctuations in aggregate output but not in aggregate markups. For this reason, in our baseline quantitative analysis, we abstract from standard aggregate productivity shocks.

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[14]In Appendix A.8 we solve for markups in the case in which each firm maximizes real profits internalizing the effect of their individual choice of output or prices on aggregate output and the real wage, thus relaxing our baseline behavioral assumption. Firms do not internalize, however, the impact that changes in profits have on the welfare of the firm's owner (Azar and Vives, 2021). We show that markups depend not only on the firm's sectoral sales share, but also on the firm's sectoral employment share as well as the firm's economy-wide share in sales and employment, which increases the computational burden of solving the model. Applying the new formula using the sales and employment shares in our baseline calibration results has a negligible impact on markup levels compared to our baseline. This is because most sectors in our data are quite small.

[15]The property is satisfied in a variety of models with variable elasticity of demand (see, e.g., the reviews in Burstein and Gopinath (2014) and Arkolakis and Morlacco (2017))

Second, the split of firm shifters V_{kit} into quality and productivity does not matter for the model implications on markups, concentration and output (for the latter, as long as deflators use quality-adjusted prices) at the firm, sector, or aggregate levels. In practice, price deflators used by statistical agencies typically do not incorporate high-frequency changes in quality. Therefore, in order to compare output in the model and data, we only consider firm-level productivity shocks and abstract from quality shocks.

1.3   Sectoral outcomes

We now describe how the model aggregates outcomes from the firm level to the sector level. We define sectoral markup as the ratio of sectoral revenues to labor payments,

$$\mu_{kt} \equiv \frac{P_{kt}Y_{kt}}{W_t L_{kt}}, \tag{6}$$

where sectoral employment is $L_{kt} = \sum_{i=1}^{N_k} L_{kit}$. Sectoral markups can be expressed as an harmonic mean (weighted by market shares) of firm-level markups,

$$\mu_{kt} = \left[\sum_{i=1}^{N_k} \mu_{kit}^{-1} s_{kit} \right]^{-1}. \tag{7}$$

Substituting the markup-market-share relationship (equation 5) under Cournot competition, we can express the sectoral markup, μ_{kt}, as a simple function of the sector's Herfindahl-Hirschman index, $HHI_{kt} = \sum_{i=1}^{N_k} s_{kit}^2$:[16]

$$\mu_{kt} = \frac{\varepsilon}{\varepsilon - 1} \left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1} \right) HHI_{kt} \right]^{-1}. \tag{8}$$

Note the positive relationship between sectoral markup and HHI takes the same form as the firm-level relationship between markup and market share in equation (5). In the same way that a firm with a large market share charges a higher markup, a sector with a large average market share, that is, a high HHI, has a high sectoral markup as long as $\varepsilon > \sigma$.[17]

Sectoral markups can be expressed as the standard ratio between sectoral price and marginal cost (i.e., the ratio of wage to sectoral productivity), $\mu_{kt} = P_{kt}Z_{kt}/W_t$. Sectoral productivity, $Z_{kt} \equiv Y_{kt}/L_{kt}$, can be expressed in terms of firm-level markups and firm shifters as

$$Z_{kt} = \frac{\left(\sum_{i=1}^{N_k} V_{kit}\mu_{kit}^{1-\varepsilon} \right)^{\frac{\varepsilon}{\varepsilon-1}}}{\sum_{i=1}^{N_k} V_{kit}\mu_{kit}^{-\varepsilon}}. \tag{9}$$

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[16]The HHI is an average of market shares, weighted by market shares themselves, and hence ranges between 0 and 1.

[17]A similar mapping between sectoral markups and concentration indices can be obtained under Bertrand competition (see Grassi, 2018).

1.4 Aggregate outcomes

We now describe how the model aggregates outcomes from the sector level to the aggregate level. We define aggregate markup as the ratio of aggregate revenues and labor payments,

$$\mu_t \equiv \frac{P_t Y_t}{W_t L_t} = \left[\sum_{k=1}^{N} s_{kt} \mu_{kt}^{-1} \right]^{-1}. \tag{10}$$

As indicated by the second equality, aggregate markups can be expressed as a harmonic weighted average of sectoral markups, where sectoral expenditure shares are determined by sectoral markups and sectoral shifters $V_{kt} \equiv A_k Z_{kt}^{\sigma-1}$,

$$s_{kt} \equiv \frac{P_{kt} Y_{kt}}{P_t Y_t} = \frac{V_{kt} \left(\mu_{kt} \right)^{1-\sigma}}{\sum_{k'} V_{k't} \left(\mu_{k't} \right)^{1-\sigma}}. \tag{11}$$

Alternatively, under Cournot, we can express the aggregate markup as a simple function of average sectoral HHI (weighted by sectoral expenditure shares) that mirrors the expressions for firm-level and sector-level markups in equations (5) and (8), respectively:

$$\mu_t = \frac{\varepsilon}{\varepsilon - 1} \left[1 - \left(\frac{\varepsilon/\sigma - 1}{\varepsilon - 1} \right) \sum_{k=1}^{N} s_{kt} HHI_{kt} \right]^{-1}.$$

The weighted average of sectoral HHIs is equal to the average market share across firms weighted by firms' expenditure share in the whole economy.[18] When this weighted-average market share in the economy is high, the aggregate markup is high.

Aggregate markups can also be expressed as the standard ratio between aggregate price and aggregate marginal cost, $\mu_t = P_t Z_t / W_t$, where aggregate productivity, $Z_t \equiv Y_t / L_t$, can be expressed in terms of sectoral markups and sectoral shifters as

$$Z_t = \frac{\left(\sum_{k=1}^{N} V_{kt} \mu_{kt}^{1-\sigma} \right)^{\frac{\sigma}{\sigma-1}}}{\left(\sum_{k=1}^{N} V_{kt} \mu_{kt}^{-\sigma} \right)}. \tag{12}$$

Finally, aggregate output and labor are given by

$$Y_t^{\eta + \frac{1}{f}} = \frac{Z_t^{1 + \frac{1}{f}}}{f_0 \mu_t} \qquad \text{and} \qquad L_t = \frac{Y_t}{Z_t}, \tag{13}$$

where the aggregate markup, μ_t, distorts the leisure/consumption choice relative to the optimal allocation.

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[18]Specifically, $\sum_{k=1}^{N} s_{kt} HHI_{kt} = \sum_{k=1}^{N} \sum_{i=1}^{N_k} s_{kt} s_{kit}^2 = \sum_{k=1}^{N} \sum_{i=1}^{N_k} \frac{P_{kt} Y_{kt}}{P_t Y_t} \frac{P_{kit} Y_{kit}}{P_{kt} Y_{kt}} s_{kit} = \sum_{k=1}^{N} \sum_{i=1}^{N_k} \frac{P_{kit} Y_{kit}}{P_t Y_t} s_{kit}.$

1.5 Summary of equilibrium

Our model aggregates outcomes in a very parsimonious manner from the firm level to the sector level, and from the sector level to the aggregate level. Here we summarize how to solve for prices and quantities as a function of time t of firm shifters, $\{V_{kit}\}$, and sectoral demand shifters, $\{A_k\}$.

Equilibrium firm-level markups and market shares, μ_{kit} and s_{kit}, are the solution to equations (4) and (5). Sectoral markups and productivities, μ_{kt} and Z_{kt}, are solved from equations (7) and (9), respectively, and sectoral expenditure shares, s_{kt}, from equation (11).

Aggregate markup, productivity, output, and employment, μ_t, Z_t, Y_t, and L_t, are solved from equations (10), (12), and (13). Setting $W_t = \bar{W}$ as the numeraire, sectoral, and aggregate price levels, P_{kt} and P_t, are given by $P_{kt} = \mu_{kt}W_t/Z_{kt}$ and $P_t = \mu_{kt}W_t/Z_t$. Sectoral output is solved from

$$Y_{kt} = A_k P_{kt}^{-\sigma} P_t^{\sigma} Y_t,$$

and sectoral employment using $L_{kt} = Y_{kt}/Z_{kt}$. Firm-level expenditures and employment, $P_{kit}Y_{kit}$ and L_{kit}, are solved from $P_{kit}Y_{kit} = s_{kit}P_{kt}Y_{kt}$ and equation (6), respectively. Finally, given a split of firm shifters V_{kit} into productivity $\{Z_{kit}\}$ and quality $\{A_{kit}\}$, firm-level output Y_{kit} and price P_{kit} are solved from equations (1) and (3), respectively.

In the following section, we use a first-order approximation to characterize the equilibrium response to firm-level shocks at the firm, sectoral, and aggregate levels.

2 Analytic results

In this section, we characterize, up to a first-order approximation, the equilibrium response of markups, prices, and output to firm-level shocks at the firm, sectoral, and aggregate levels.[19]

We first introduce a first-order approximation to solve for changes in firm-level markups and market shares in a sector. We then develop expressions for changes in prices, markups, and output in response to firm-level shocks, first at the sector level and then at the aggregate level. We provide expressions for asymptotic covariances between markup and output changes at different aggregation levels under the additional assumption that firm-level shocks are i.i.d. and equally distributed across firms with variance $\sigma_v^2 \equiv \mathbb{V}ar\left[\widehat{V}_{kit}\right]$. We focus on the case of Cournot competition, and present results under Bertrand in the appendix. We highlight the role of variable markups versus constant markups in shaping markup cyclicality, as well as the impact of variable markups on aggregate output volatility. We return to these formulas in our quantitative analysis in section 4.3.

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[19]We thank Dmitry Mukhin for his valuable input in deriving these analytic results.

2.1 Firm-level outcomes

Consider an initial equilibrium in sector k with market shares $\{s_{ki}\}$ and markups $\{\mu_{ki}\}$ where, for simplicity, we omit time subscripts in the initial equilibrium. Taking a first-order approximation of the expressions of market share (equation 4) and of firm-level markup (equation 5), changes in the equilibrium market shares and markups are the solution to the following system of equations

$$\widehat{s}_{kit} = \widehat{V}_{kit} + (1 - \varepsilon)\,\widehat{\mu}_{kit} - \sum_{i'=1}^{N_k} s_{ki'} \left(\widehat{V}_{ki't} + (1 - \varepsilon)\,\Gamma_{ki'}\widehat{s}_{ki't} \right), \tag{14}$$

$$\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}. \tag{15}$$

Variables with hats denote log differences at time t relative to the initial equilibrium, that is, $\widehat{V}_{kit} \equiv \log V_{kit} - \log V_{ki}$, and Γ_{ki} denotes the *markup elasticity* with respect to market share for firm i in sector k, evaluated at the initial equilibrium.

Markup elasticities under Cournot are, by equation (5),

$$\Gamma_{ki} \equiv \frac{\partial \log \mu_{ki}}{\partial \log s_{ki}} = \frac{\left(\frac{\varepsilon}{\sigma} - 1 \right) s_{ki}}{\varepsilon - 1 - \left(\frac{\varepsilon}{\sigma} - 1 \right) s_{ki}}.$$

As discussed above, if $\varepsilon > \sigma$, markups are increasing in market shares i.e., $\Gamma_{ki} \geq 0$, with strict inequality if $s_{ki} > 0$. Moreover, markup elasticities are also increasing in market shares. This property whereby markup elasticities are increasing in market shares is satisfied by a variety of demand models with variable elasticity, as discussed in, for example, Burstein and Gopinath (2014) and Arkolakis and Morlacco (2017).

We now introduce pass-through elasticities, which are not required to solve for sectoral market shares and markups but, nevertheless, we use in our analytic results that follow. We choose the wage as the numeraire, without loss of generality. Changes in firm-level prices (relative to the wage) at time t are given by $\widehat{P}_{kit} = -\widehat{Z}_{kit} + \widehat{\mu}_{kit}$. Combined with equations (15) and $\widehat{s}_{kit} = \widehat{A}_{kit} + (1 - \varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt} \right)$, we obtain

$$\widehat{P}_{kit} = \alpha_{ki} \left(-\widehat{Z}_{kit} + \Gamma_{ki}\widehat{A}_{kit} \right) + (1 - \alpha_{ki})\,\widehat{P}_{kt}, \tag{16}$$

where α_{ki} is the *pass-through rate* governing how firm-level prices respond to idiosyncratic shocks (for given changes in sectoral prices, $\widehat{P}_{kt}$),

$$\alpha_{ki} = \frac{1}{1 + (\varepsilon - 1)\,\Gamma_{ki}}. \tag{17}$$

Conversely, $1 - \alpha_{ki}$ governs how prices respond to changes in sectoral price (due to variable

markups).[20] Because markup elasticities are increasing in market shares (if $\varepsilon > \sigma$), pass-through rates are decreasing in market shares.[21] To isolate the role of changes in markups in response to shocks, we consider the case in which markups are fixed at the initial equilibrium levels, imposing $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$.

2.2 Sectoral outcomes

In this subsection, we characterize how sectoral prices, markups, and output respond to firm-level shocks, and provide expressions for variances and covariances of markup and output changes over long realizations of shocks.

Sectoral prices As a first step in understanding changes in sectoral output, we characterize changes in sectoral prices (relative to the the numeraire, i.e., wage), which are related to sectoral output by CES demand, $Y_{kt} = A_k P_{kt}^{-\sigma} P_t^\sigma Y_t$.

Taking a first-order approximation of the sectoral price definition (2) and using firm-level price changes (16), log changes in sectoral prices can be expressed as a weighted average of firm shifters,

$$\widehat{P}_{kt} = -\frac{1}{\varepsilon - 1} \frac{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \widehat{V}_{kit}}{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki}}, \tag{18}$$

where the weights are given by the product of market shares, s_{ki}, and pass-through rates, α_{ki}. Because $\varepsilon \geq 1$, sectoral prices fall in response to an increase in firm shifter.

To understand how sectoral price changes are shaped by pass-through rates, note that if $\alpha_{ki} = \alpha_k$, $\widehat{P}_{kt}$ is independent of α_k for given market shares s_{ki} in the initial equilibrium. That is, the response in sectoral price is identical to that if markups were fixed at their initial level ($\alpha_{ki} = 1$). Intuitively, as pass-through α_k falls, the larger markup change by a firm to an own shock is exactly offset by a larger change in markup, in the opposite direction, of its competitors.

With heterogeneity in pass-through rates, because α_{ki} is decreasing in s_{ki}, a single value $\bar{s}_k^p$ exists such that a positive shock to firm i with $s_{ki} > \bar{s}_k^p$ results in a smaller reduction in sectoral prices than if markups were fixed at their initial level. Intuitively, firm i's increase in markup more than offsets the markup decrease of its competitors. Conversely, a positive shock to firm i with $s_{ki} < \bar{s}_k^p$ results in a larger reduction in sectoral prices than if markups were fixed at their initial level.[22]

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[20]In the appendix, we provide expressions for the elasticity of market shares with respect to firm-level shifters and for the variance of market shares.

[21]We can further solve for $\widehat{P}_{kit}$ using $\widehat{P}_{kt} = s_{ki}\widehat{P}_{kit} + (1 - s_{ki})\widehat{P}_{k-it}$, where $\widehat{P}_{k-it}$ is the competitors' price index defined in Amiti et al. (2019). We can rewrite (16) as $\widehat{P}_{kit} = \tilde{\alpha}_{ki}\left(-\widehat{Z}_{ki} + \Gamma_{ki}\widehat{A}_{kit}\right) + (1 - \tilde{\alpha}_{ki})\widehat{P}_{k-it}$, where $\tilde{\alpha}_{ki} = \frac{\alpha_{ki}}{1-(1-\alpha_{ki})s_{ki}}$, which is a U-shaped function of market shares s_{ki}.

[22]The threshold $\bar{s}_k^p$ is defined implicitly by $\alpha_k(\bar{s}_k^p) = \sum_{i=1}^{N_k} s_{ki}\alpha_{ki}$.

From equation (18), the asymptotic variance of price changes in sector k assuming firm-level shifters are i.i.d. with common variance σ_v^2 is

$$\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon - 1}\right)^2 \sum_{i=1}^{N_k}\left(\frac{\alpha_{ki}s_{ki}}{\sum_{i'}\alpha_{ki'}s_{ki'}}\right)^2. \tag{19}$$

If markups are fixed at their initial level (or, more generally, if $\alpha_{ki} = \alpha_k$), this variance is proportional to the sectoral HHI, as in Gabaix (2011): $\mathbb{V}ar\left[\widehat{P}_{kt}\right] = \left(\frac{\sigma_v}{\varepsilon-1}\right)^2 \sum_{i=1}^{N_k} s_{ki}^2$. Comparing this expression with (19), we note $\mathbb{V}ar\left[\widehat{P}_{kt}\right]$ is lower under variable markups than under constant markups if and only if the variance of $\frac{\alpha_{ki}s_{ki}}{\sum_{i'}\alpha_{ki'}s_{ki'}}$ is lower than the variance of s_{ki}. Because α_{ki} is decreasing in s_{ki}, this condition is satisfied if $s_{ki}\alpha_{ki}$ is increasing in s_{ki} (see condition A10 in appendix A.3). Intuitively, under this condition, pass-through rates are lower for larger firms, effectively reducing the weight of large firm shocks in the price index (with similar effects on volatility as a decline in the HHI).

Sectoral markups   Changes in sectoral markups, defined in equation (7), can be decomposed into changes in markups *within* firms and the reallocation of expenditures *between* firms with heterogeneous markups:

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki}\frac{\mu_k}{\mu_{ki}}\left(\widehat{\mu}_{kit} - \widehat{s}_{kit}\right). \tag{20}$$

In appendix A, we derive the following expression for changes in sectoral markups:[23]

$$\widehat{\mu}_{kt} = 2\left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)\mu_k \sum_{i=1}^{N_k} s_{ki}\alpha_{ki}\left[s_{ki} - \frac{\sum_{i'} s_{ki'}^2\alpha_{ki'}}{\sum_{i'} s_{ki'}\alpha_{ki'}}\right]\widehat{V}_{kit}. \tag{21}$$

The following proposition states that a positive shock to firm i results in an increase in the sectoral markup if and only if firm i is sufficiently large in its sector.

Proposition 1 *Consider a positive shock to firm i in sector k, $\widehat{V}_{kit} > 0$. Then, under Cournot competition, sector k markup increases, $\widehat{\mu}_{kt} > 0$, if and only if $s_{ki} > \sum_{i'} s_{ki'}^2\alpha_{ki'}/\sum_{i'} s_{ki'}\alpha_{ki'}$.*

Intuitively, recall from equation (20) that changes in sectoral markups reflect changes in firm-level markups (within term) and between-firm reallocation (between term). Consider first the within term. A positive shock to firm i raises firm i's markup and reduces it for competing firms. The former dominates if firm i is large, whereas the latter dominates if firm i is small. Consider now the between term. A positive shock to firm i reallocates market shares towards firm i, increasing the sectoral markup if firm i's markup is sufficiently high (or, equivalently, if

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[23]Ex-ante firm heterogeneity is a necessary condition for sectoral markups to change in response to firm-level shocks. To see this, if s_{ki} and μ_{ki} are equal across all firms in sector k, equations (15), (20), and $\sum_{i=1}^{N_k} s_{ki}\widehat{s}_{ki} = 0$ imply that $\widehat{\mu}_{kt} = 0$.

its market share is sufficiently large). Therefore, the within and between terms push the sectoral markup in the same direction.

The "2" in front of (21) reflects the fact that the magnitude of the within term is equal to the magnitude of the between term (and hence each accounts for 50% of changes in sectoral markups). A change in parameters (e.g., an increase in $\varepsilon - \sigma$) that increases the sensitivity of markups to firm-level shocks (increasing the within term) also increases the dispersion of markups across firms (increasing the between term). In appendix A, we show this 50-50 within/between decomposition of changes in sectoral markups under Cournot competition holds globally not only up to a first order.

How do changes in sectoral markups compare in the specification with variable markups versus the specification with constant markups in which sectoral markups change only due to between-firm reallocation? If firm-level markups are fixed at their initial level (setting $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$), changes in sectoral markups in equation (20) are:

$$\widehat{\mu}_{kt} = \sum_{i=1}^{N_k} s_{ki} \left(1 - \frac{\mu_k}{\mu_{ki}} \right) \widehat{V}_{kit}. \tag{22}$$

In response to a positive shock to firm i, sectoral markups increase if and only if $\mu_{ki} > \mu_k$.

In general, we cannot easily compare (22) and (21). To make analytic progress, in appendix A, we restrict the extent of ex-ante firm heterogeneity to two types and provide a simple sufficient condition for sectoral markups to change by more (and display a higher variance) under variable markups than under constant markups. Intuitively, changes in sectoral markups can be smaller under variable markups than under constant markups because the larger response of sectoral markups due to changes in firm-level markups is more than offset by a smaller extent of between-firm reallocation due to incomplete pass-through.

To summarize, even though changes in sectoral markups under variable markups are twice as large as the between-firm reallocation term for any firm-level shocks, variable markups do not necessarily magnify changes in sectoral markups relative to the model specification with constant markups, because incomplete pass-through mutes the extent of between-firm reallocation

Covariance between sectoral prices and sectoral markups   Recall from previous results that in response to a positive shock to firm i in sector k, the sectoral price falls, whereas sectoral markup can increase or decrease depending on the firm's initial markup. In finite samples, comovement can be positive or negative depending on which firms are shocked. We now calculate the asymptotic covariance between sectoral price and markup changes, which shapes the covariance between sectoral output and markup that we examine below.

First, to build intuition, in the case of constant markups,

$$\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] = -\frac{1}{\varepsilon - 1}\sum_{i=1}^{N_k} s_{ki}^2\left[1 - \frac{\mu_k}{\mu_{ki}}\right] \times \sigma_v^2. \tag{23}$$

Thus, sectoral markups and prices are negatively correlated as long as large firms within sector charge higher markups. Intuitively, shocks to small firms induce a positive comovement, whereas shocks to large firms induce a negative comovement. Overall, comovement is negative because shocks to large firms induce larger changes in sectoral price than shocks to small firms.

With variable markups, using the expressions for the change in sectoral price (18) and markup (21),

$$\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] = -\left(\frac{2\mu_k}{\varepsilon - 1}\right)\left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}\sum_{i=1}^{N_k}\left[\frac{s_{ki}^2\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}} - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} \times \sigma_v^2. \tag{24}$$

Therefore, when $\varepsilon > \sigma$, sectoral prices and markups comove negatively in long samples if and only if

$$\sum_{i=1}^{N_k}\left[\frac{s_{ki}^2\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}^2\alpha_{ki'}} - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}} > 0. \tag{25}$$

If firms are ex-ante homogeneous, equation (25) holds with equality and sectoral markups are constant over time. If firms are heterogeneous in the initial equilibrium, inequality (25) may or may not hold. The following proposition, proven in the appendix, states that if pass-through rates do not fall too strongly with market shares, inequality (25) holds, so sectoral prices and markups comove negatively.

Proposition 2 *Under Cournot competition, if firms are ex-ante heterogeneous and $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, sectoral markup and price comove negatively, $\mathbb{C}ov\left[\widehat{\mu}_{kt}, \widehat{P}_{kt}\right] < 0$.*

In appendix A.3 we show that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} provided that market shares are not too large. Intuitively, the condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} implies, by equation (18), that sectoral prices are more responsive to large firm shocks than to small firm shocks (i.e., the lower pass-through rate by large firms does not fully offset their higher weight in the price index). The fact that sectoral markups increase in response to large firm shocks and decrease in response to small firm shocks implies a negative covariance between sectoral price and markup, as in the case of constant markups.

Covariance between sectoral output and markups In appendix A, we derive the following expression for changes in sector k output in response to sector k shocks:

$$\widehat{Y}_{kt} = -\left[\sigma\left(1-s_k\right) + \left(\frac{f+1}{f\eta+1} + \left(\frac{\sigma-1}{f\eta+1}\right)\left(1-\frac{\mu}{\mu_k}\right)\right)s_k\right]\widehat{P}_{kt} + \frac{s_k\mu}{\mu_k}\frac{\widehat{\mu}_{kt}}{f\eta+1}. \tag{26}$$

A sufficient condition for sectoral output and price to move in opposite directions is that sector k is small in the aggregate ($s_k \to 0$) or that disutility of labor is linear ($f \to \infty$). In this case, the previous results on sectoral price apply immediately to sectoral output (with the opposite sign).[24] Specifically, in response to a positive shock to firm i in sector k, sectoral output increases whereas sectoral markup can increase or decrease depending on the firm's initial markup.

Taking into account a long sequence of firm shocks in sector k, the covariance between changes in sectoral output and sectoral markup is:

$$\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right] = -\left[\sigma\left(1-s_k\right) + \frac{f+1+(\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)}{f\eta+1}s_k\right]\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right] + \frac{s_k\mu}{\mu_k}\frac{1}{f\eta+1}\mathbb{V}ar\left[\widehat{\mu}_{kt}\right],$$

where $\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right]$ is defined above. The following proposition provides sufficient conditions for pro-cyclical sectoral markups with respect to sectoral output.

Proposition 3 *Under the conditions of Proposition 2, sectoral markup and sectoral output co-move positively, $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right] > 0$, if at least one of these three conditions holds: (i) $s_k \to 0$, (ii) $f \to \infty$, (iii) $\sigma \to 1$. If all three conditions (i)-(iii) are violated, $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kt}\right] > 0$ as long as sectoral markup μ_k is not too low relative to aggregate markup.*

In our empirical analysis, we also consider the cyclicality between sector output and firm-level markups. In appendix A, we show that, for the case of $f \to \infty$, the covariance between changes in firm i markup and sector k output is

$$\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right] = \left(\sigma\left(1-s_k\right) + \eta^{-1}s_k\right)\frac{\alpha_{ki}\Gamma_{ki}}{(\epsilon-1)\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\left[s_{ki}\alpha_{ki} - \frac{\sum_{i'=1}^{N_k}(s_{ki'}\alpha_{ki'})^2}{\sum_{i'=1}^{N_k}s_{ki'}\alpha_{ki'}}\right]\times\sigma_v^2. \tag{27}$$

The following proposition states that firm-level markups are procyclical for large firms and counter-cyclical for small firms:

Proposition 4 *If $s_{ki}\alpha_{ki}$ is increasing in s_{ki} and $f \to \infty$, firm-level markups and sectoral output comove positively, $\mathbb{C}ov\left[\widehat{Y}_{kt},\widehat{\mu}_{kit}\right] > 0$, if and only if $s_{ki} > \bar{s}_k^\mu$, and comove negatively if and only if $s_{ki} < \bar{s}_k^\mu$, where $\bar{s}_k^\mu$ is defined by the condition that the square bracket in (27) is equal to 0.*

Intuitively, firm-level markups are positively correlated with sectoral output in response to

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[24]If f finite and sector k is sufficiently large in the aggregate, it is *possible* that sectoral output and price both fall in response to positive sector k firm level shocks if sectoral markup μ_k is very low relative to the aggregate markup and/or if sector k markup falls substantially when the sectoral price falls.

own-shocks and negatively correlated in response to competitors' shocks. Because large firms have a disproportionate impact on sectoral price and output (if $s_{ki}\alpha_{ki}$ is increasing in s_{ki}), firm-level markups are pro-cyclical for large firms and counter-cyclical for small firms.[25]

Before presenting the aggregate results, we briefly discuss our model's implications for changes in markups when some prices are nominally rigid.

Discussion of firm and sector-level markups with rigid prices   Whereas in this paper we study markup fluctuations under flexible prices, markups can also fluctuate if costs change and prices are nominally rigid. Consider the specification under Bertrand competition, and suppose that the price of firm i in sector k at time t is stuck at $\bar{P}_{kit}$ before the shocks hit.[26] For a sticky price firm, the markup is $\mu_{kit} = Z_{kit}\bar{P}_{kit}/W_t$. For flexible price firms (which do not anticipate that that their price may be stuck in the future), markups are given by (5). Market shares for all firms are determined by the system of equations (4).

In response to productivity shocks to flexible price firms, the sign of the change in sectoral markups depends on the relative markup level of the shocked firms, as in the baseline model. Consider now an increase in productivity of a rigid-price firm and suppose that the change in nominal wage is negligible. The markup of the shocked rigid-price firm rises mechanically, while markups of other firms remain unchanged (since prices and thus market shares do not change). Hence, the sectoral markup rises irrespective of whether the shocked firm is small or large. This force strengthens pro-cyclical sectoral markups in comparison to the flexible-price model.

Consider now a uniform decline in marginal costs for all firms (productivity rises relative to the nominal wage). For firms with rigid price, markups rise. For firms with flexible price, markups also rise since these firms lower their price and increase market share relative to sticky price firms. Hence, markups rise for all firms. There are additional compositional effects on the sectoral markup as market shares shift towards flexible price firms. This composition effect increases the sectoral markup if flexible price firms charge higher markups.[27] Whether the increase in markups is pro-cyclical or countercyclical depends on the source of the movement in marginal cost. In response to an increase in productivity for all firms, markups and output rise. This force provides another reason for pro-cyclical markups relative to the flexible price baseline where, recall, sectoral shocks leave markups unchanged. In response to contractionary monetary policy that reduces marginal cost for all firms, markups rise but output falls,

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[25]The cutoff $\bar{s}_k^\mu$ differs from the cutoff defined in Proposition 1, because the condition in Proposition 1 is based on a shock to one firm only, whereas the asymptotic covariance in Proposition 4 takes into account shocks to all firms in the sector.

[26]Here for simplicity we take $\bar{P}_{kit}$ as given and do not study how firms choose their reset price. For a detailed analysis of sticky prices in a dynamic environment with oligopolistic competition, see Mongey (2017) and Wang and Werning (2022).

[27]For evidence that prices are more flexible for large firms (which in our model charge higher markups), see Goldberg and Hellerstein (2009) and D'Acunto et al. (2018).

implying countercyclical markups. Our flexible-price baseline abstracts from this well-studied source of countercyclical markups.

2.3 Aggregate outcomes

We now describe how changes in sectoral markup, price, and output that we characterized above shape changes in aggregate price (i.e., the inverse of the real wage given that the wage is the numeraire), markup, productivity, and output. We provide expressions for sectoral and aggregate markup cyclicality with respect to aggregate output, which we consider in our empirical analysis. We also examine how the variance of aggregate output compares under variable markups and constant markups.

Up to a first order, changes in the aggregate price are $\widehat{P}_t = \sum_k s_k \widehat{P}_{kt}$. Based on our results above, any positive firm-level shock in sector k reduces the corresponding sectoral price and therefore reduces the aggregate price (or increases the real wage) proportionately to the share in expenditures of sector k. Whether the real wage increases more or less under variable markups relative to constant markups depends, as discussed above, on the shocked firm's relative size in its sector.

Changes in aggregate markup can be decomposed into a within-sector markup term and a reallocation term, analogous to the decomposition of sectoral markups in equation (20):

$$\widehat{\mu}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} + (1 - \sigma) \sum_k s_k \left(1 - \frac{\mu}{\mu_k} \right) \widehat{P}_{kt}. \tag{28}$$

In response to a positive shock to a firm in sector k, aggregate markup can increase or decrease. The first (within) term in (28) is positive if the shocked firm is relatively large (and sets a higher markup) in sector k. The second (between) term in (28) is positive, when $\sigma > 1$, if sector k has a relatively high markup relative to the aggregate markup.

Changes in aggregate productivity, using $\widehat{Z}_t = \widehat{\mu}_t - \widehat{P}_t$, can be expressed in terms of changes in sectoral markups and prices as

$$\widehat{Z}_t = \sum_k s_k \frac{\mu}{\mu_k} \widehat{\mu}_{kt} - \sum_k s_k \left[1 + (\sigma - 1) \left(1 - \frac{\mu}{\mu_k} \right) \right] \widehat{P}_{kt}. \tag{29}$$

Recall that in response to positive firm-level shocks, the sectoral price decreases ($\widehat{P}_{kt} < 0$). Aggregate productivity typically increases, but can decrease if shocked firms are relatively small in their sector (such that the sectoral markup falls) or belong to low-markup sectors and $\sigma > 1$.

Finally, by equation (13), changes in aggregate output are

$$\widehat{Y}_t = (f^{-1} + \eta)^{-1} \left[f^{-1} \widehat{Z}_t - \widehat{P}_t \right]. \tag{30}$$

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With inelastic labor supply ($f \to 0$), $\widehat{Y}_t = \widehat{Z}_t$. With linear disutility of labor ($f \to \infty$), the aggregate productivity term drops, so $\widehat{Y}_t = -\eta^{-1} \sum_k s_k \widehat{P}_{kt}$. A positive firm-level shock in sector k reduces the sectoral price and increases aggregate output. Based on the discussion above on the role of variable markups for the response of sectoral prices, the increase in aggregate output is smaller under variable markups compared to constant markups if and only if the shocked firm has a high market share.

Variance of aggregate output   The variance of aggregate output (when $f \to \infty$) is

$$\mathbb{V}ar\left[\widehat{Y}_t\right] = \eta^{-2} \sum_k s_k^2 \mathbb{V}ar\left[\widehat{P}_{kt}\right] = \frac{\sigma_v^2}{\eta^2 (\varepsilon - 1)^2} \sum_k s_k^2 \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}}\right)^2, \qquad (31)$$

where the second equality used equation (19). Based on the discussion after equation (19) above, aggregate output is less volatile under variable markups than under constant markups when pass-through rates are decreasing in size, effectively reducing the weight of large firms in the price index (with similar effects on volatility as a reduction in market-share concentration).

In appendix A, we provide an expression for the variance of aggregate output without imposing $f \to \infty$, as well as for the variance of aggregate markups.

Covariance between aggregate output and markups   We first calculate the covariance between aggregate output and sector k markup, which is one of the measures of cyclicality in our empirical analysis. When calculating this covariance, we use the fact that sector k markups are affected only by shocks to sector k firms and not by shocks to firms in other sectors. We can thus express this covariance as

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] = \mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right] + \sigma (1 - s_k) \mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right]. \qquad (32)$$

The following proposition states that the covariance between aggregate output and sector k markups is positive:[28]

Proposition 5   *Under the conditions of Proposition 3, $\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] > 0$.*

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[28]To prove the first inequality in Proposition 5, we write equation (32) as

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] = -s_k \left[\frac{f + 1 + (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right)}{f\eta + 1}\right] \mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{s_k \mu}{\mu_k} \frac{1}{f\eta + 1} \mathbb{V}ar\left[\widehat{\mu}_{kt}\right].$$

Note also that under the conditions of Proposition 3, $\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] \leq 0$ so $\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_{kt}\right] \leq \mathbb{C}ov\left[\widehat{Y}_{kt}, \widehat{\mu}_{kt}\right]$, where the inequality holds strictly if the economy has more than one sector (i.e. $s_k < 1$). The fact that the covariance between sectoral markups and aggregate output is lower than that between sectoral markups and sectoral output does not extend to correlations because, for some sectors, the variance of aggregate output is smaller than the variance of sectoral output.

Next, we calculate the covariance between aggregate output and aggregate markups (when $f \to \infty$):

$$\mathbb{C}ov\left[\widehat{Y}_t, \widehat{\mu}_t\right] = -\frac{\mu}{\eta}\sum_k \frac{s_k^2}{\mu_k}\mathbb{C}ov\left[\widehat{P}_{kt}, \widehat{\mu}_{kt}\right] + \frac{\sigma - 1}{\eta}\sum_k s_k^2\left(1 - \frac{\mu}{\mu_k}\right)\mathbb{V}ar\left[\widehat{P}_{kt}\right]. \tag{33}$$

The first term in (33) is positive if sectoral markups and sectoral prices comove negatively, which we discussed above. The second term in (33) is positive unless larger sectors have relatively lower markups.

So far, we have calculated measures of markup cyclicality considering only i.i.d firm-level shocks. In our quantitative analysis, we also allow for aggregate productivity shocks to firms in all sectors. In our model, in which firm-level markups are functions of market shares, markups do not respond to aggregate shocks. Therefore, incorporating aggregate shocks leaves the covariance of aggregate markups and output unchanged but decreases the correlation, because the volatility of aggregate output increases with these shocks.

From these theoretical results, we see the sign of markup cyclicality depends on the level of aggregation, market structure within and across all industries, and the set of shocked firms. Moreover, the sign and magnitude of covariances in finite samples may differ from those of the asymptotic covariances we derived.

In what follows, we calibrate the model to match salient features of the French firm-level data. We evaluate quantitatively its implications for the cyclicality of markups, as well as its ability to generate aggregate fluctuations in output and markups in response to idiosyncratic firm-level shocks.

3   Data, Estimation, and Calibration

For the remainder of this paper, we use the model above as a data-generating process from which we simulate firm-level outcomes then aggregated into sector and aggregate time series. We then proceed to compare the resulting model-implied moments to their empirical counterparts. In this section, we describe how we use French administrative firm-level data to parametrize our model and provide the empirical moments of interest. We start by describing the data and how we estimate markups. We then describe how we parametrize the firm shock process and how we calibrate the model. Appendix B provides additional information on the data and the procedures to estimate production functions and markups. Appendix C reports robustness of our empirical results for different choices of data selection and markup estimation.

3.1 Data

Our empirical analysis deploys French firm-level data between 1994 and 2019. We use administrative sources for income statements and balance sheets, and a survey for information on quantity and price.

The firm-level information (except for quantity and price) comes from two administrative datasets: the FICUS data covering the period 1994-2007 and the FARE dataset covering the period 2008-2019. The datasets cover the universe of French firms and originate from the French tax administration that collects yearly tax statements for each firm, including income statements, balance-sheet, and demographic information. The Institut National de la Statistique et des Etudes Economiques (INSEE) uses these statements to construct the FICUS-FARE datasets. We assign firms to sectors according to the Nomenclature d'Activités Française (NAF2008) five-digit classification, which is a French industry classification similar to the NACE Rev. 2 industry classification at four-digit.[29]

We use a subset of the variables available in the FICUS-FARE dataset: total firm revenues, wage bill (sum of wages and social security payments), capital (measured by fixed assets), and expenditures on both material and service inputs. Our baseline measure of materials (which is our choice of variable input in the estimation of markups) is the sum of expenditures on materials and merchandises (ACHAMPR and ACHAMAR, respectively) net of changes in stocks of materials (VARSTMP for materials and VARSTMA for merchandise). The variable ACHAMPR is defined as "everything that the firm purchases in order to be transformed," and ACHAMAR is defined as "everything that the firm purchases to be sold as is." VARSTMP and VARSTMA are defined analogously for changes in stocks. We consider as a separate input expenditures on service inputs (AUTACH), including research expenditures, outsourcing costs, and external personnel cost (including temporary workers). We use GDP deflators and two-digit sector-price indices provided by EU-KLEMS.

We construct firm-level quantity as firm-level revenues deflated by price (calculated as described below) using the survey Enquête Annuelle de Production (EAP) conducted by INSEE. This survey covers the universe of large firms (with at least 20 employees or 5 million euro annual revenues) and a representative sample of small firms for a subset of two-digit sectors, mostly in manufacturing, in the period 2009-2019.[30] For each firm, this dataset provides in-

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[29]In 2008, the NACE and NAF industry classification changed. To construct a panel of firms between 1994 and 2019 with a consistent industry classification over time, we proceed as follows. For firms for which the old and new industry codes are observed, we apply the new code to all years. For firms for which only observe one industry code on either side of 2008, we assign the code that is most frequently associated with the observed industry code (using the sample of firms where we do observe the two sectoral codes). We thank Isabelle Mejean for sharing the code to help merge the FICUS and FARE datasets.

[30]The two-digit sectors according to the NACE rev 2 codes, covered by our EAP sample are 08, 13, 14, 15 ,16 ,17 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 43 and 46, where the last two are non-manufacturing sectors ("specialized construction activities" and "wholesale trade, except of motor vehicles and motorcycles" respectively)

formation on revenues and quantities by product, where product is a combination of a 8-digit code and a unit of account.[31] We drop around one-third of firm-products without quantity data. For each firm and product, we calculate price as the ratio of revenues to quantity sold. We follow De Ridder et al. (2022) and Aghion et al. (2023) in standardizing prices by dividing them by the quantity-weighted average price of the same product across firms in the sample in the same year. We do this because firms produce different goods in different units (e.g. kilograms and liters) and aggregation requires homogenous units. Our measure of firm price in a given year is the revenue-weighted average of standardized prices across the products sold by the firm. Firm-level quantity is defined as the ratio of firm-level revenues to firm-level price.

We consider two samples: (i) firms with price and quantity information in the EAP dataset, that we use to estimate two-digit sector-level production functions used in the markup calculation and (ii) all firms that belong to sectors covered by the EAP dataset, that we use to calculate our measures of markup cyclicality.

For the first sample, which covers the period 2009-2019, we keep firms with more than 2 employees and with positive value added, revenue, materials, services expenditure, wage bill, capital, and price. We winsorize these variables by sector at the 1% level. We end up with $220,733$ firm-year observations across 11 years and 22 two-digit sectors. For the second sample, which covers the period 1994-2019, we keep firms with positive value added, revenue, materials, services expenditure, wage bill, capital, and markups.[32] Finally, we keep firms that are government-owned earlier on in our sample because most of them switched to private ownership during the period we consider.[33] We end up with a firm-level panel that covers 26 years, 22 two-digit sectors and 275 five-digit sectors.

Table A1 in the Appendix B.1 displays summary statistics for both the estimation sample (Panel A) and the baseline sample used to measure markup cyclicality (Panel B).The number of observations in the baseline sample is about 40 times larger than in the estimation sample. This is due to both the smaller number of years and firms covered by the EAP survey. Furthermore, firms in the estimation sample are larger than firms in the baseline sample since the EAP survey focuses on large firms. On the baseline sample, we compute firm-level market share as the share of revenue within a five-digit sectors. The average market share across all firms and years, defined as revenue of a firm divided by total revenue of firms in the same five-digit sector, is very low at about 0.07%. However, the distribution of market shares is highly skewed, with the top 0.01% firms having a market share of above 38%.

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[31]Examples of units of accounts are kilograms, tonnes, or pieces. We define a product as a combination of a unit of accounts and product code because firms that use different units of accounts for the same product code might produce relatively heterogeneous goods.

[32]We drop firms in a handful of five-digit sectors where for some years there are no firms with a positive value-added. For these five-digit sectors we are unable to construct sector-level output.

[33]Information about government-owned firms can be found in the variable APPGR of FICUS-FARE, which is available only before 2009. Government-owned firms represented 0.12% of the total number of firms in 1994 and 0.05% in 2008. Over the same period, their share of revenue fell from 7.2% to 4.2%.

Although this dataset is extremely rich, it misses some important information that limits the extent of our analysis. First, we do not use information on imports and exports in the corresponding sector. Specifically, when we compute market share as the ratio of a firm's revenue relative to the sum of all French firms' revenue in this sector, we do not take into account the sales of foreign firms in this market. Moreover, when we estimate markups, we do not exclude sales to foreign countries, because we do not know the share of inputs expenditure accounted for by exports.

Second, because firm-level revenues in our dataset are reported at the national level, we do not have information on revenues at the local level. This limitation is important for non-tradeable goods, whose markets are most likely local.[34] Because our definition of a market is at the national level, for non-tradable goods, we likely underestimate the concentration in the local market relevant for the firm.

3.2 Markup Estimation

To compute firm-level markup, we need estimates of the production function and the ratio of firm revenues to expenditures on a variable input. We estimate the former for each two-digit sector based on the sample of firms in the estimation sample which includes quantity information. We measure the latter for all firms in the baseline sample which, recall, has a larger coverage than the estimation sample.

We use our markup estimates for two purposes. First, we calculate measures of markup cyclicality in the data, which we compare with markup dynamics implied by our model. Second, when we calibrate the model, we target the relationship between sectoral markups and HHI.

Our empirical framework to estimate markups in the data is more general than our theoretical framework described above, where labor was the only factor of production. Specifically, we introduce materials, capital and service in addition of labor as factors of production, some of which may be subject to adjustment frictions.[35] However, we assume that materials is a variable input that is not subject to adjustment cost or intertemporal choices. Following Hall (1988) and De Loecker and Warzynski (2012), for the variable input material, M, the first-order condition in the cost-minimization problem by firm i in sector k implies

$$\mu_{kit} = \theta_{kit}^M \frac{P_{kit} Y_{kit}}{P_{kit}^M M_{kit}}, \tag{34}$$

where $P_{kit}^M M_{kit}$ denotes expenditure on input M by firm i in sector k, $P_{kit} Y_{kit}$ is revenues, and

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[34]See Rossi-Hansberg et al. (2020) for a study of diverging local and national market-concentration trends or Smith and Ocampo (2023) for the evolution and consequences of market-concentration trends in the US retail sector.

[35]We maintain the assumption that firms are price takers in input markets. Morlacco (2019) relaxes this assumption to estimate markdowns on inputs.

θ_{kit}^M is the output elasticity with respect to input M. To measure markups at the firm level, we require the ratio of material expenditures to revenues — which is available for all firms in our baseline sample — and the output elasticity with respect to materials.

To estimate output elasticities with respect to materials, we combine data on input usage at the firm level and estimates of the production function. We assume that firms combine the four inputs (labor, capital, materials, and services) according to a flexible translog production function. All firms in the same two-digit sector share the same production-function parameters and — constrained by the availability of two-digit-level price indices for intermediate inputs — inputs are homogeneous across firms within sector. The output elasticity with respect to materials of firm i in market k at time t is equal to $\theta_{kit}^M = \beta_m + 2\beta_{m^2} m_{kit} + \beta_{ml} l_{kit} + \beta_{mo} o_{kit} + \beta_{mk} k_{kit}$ where the β_{xy} are the parameter of the production functions and $m_{kit}, l_{kit}, o_{kit}$ and k_{kit} are respectively the firm-level log materials, labor, service and capital usage. Note that output elasticities differ across firms within two-digit sectors even if these firms have the same production function parameters β.

To estimate the production function parameters, we implement a two-stage iterative generalized method of moments (GMM) following De Ridder et al. (2022) which builds on Olley and Pakes (1996), Levinsohn and Petrin (2003), and Ackerberg et al. (2007, 2015). Here we provide an overview of our approach, and in Appendix B.2 we provide additional details. The first stage of this method controls for unobserved productivity by using conditional demand of material input. As discussed in Doraszelski and Jaumandreu (2019) and De Ridder et al. (2022), under imperfect competition of the form considered in this paper, this first-stage requires additional controls, namely market share and firm-level price. In the second stage, we implement a dynamic panel estimator using GMM. Importantly, in both stages we use quantity data as our output measure, thus we can only implement this approach in our estimation sample of firms.[36] Table A1 provides descriptive statistics of our firm-level markup estimates. The distribution of markups is quite skewed, with a median of 1.21, a mean of 1.39 and a top quartile of 1.77.

3.3 Calibration

In this section, we describe how we parametrize the model to match salient features of the French data. We first introduce the firm-level productivity stochastic process, which follows a discrete Markov chain giving rise to random productivity growth as in the literature on firm dynamics (see, e.g., Luttmer, 2010). We then describe how we target the size and concentration of each of the 275 five-digits sectors, together with other moments from our data and the literature. Finally, we discuss the model's implications for other empirical moments that are not based on our estimates of markups.

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[36]Since we use quantity as a measure of output, our methodology is not subject to biases due to the use of revenues. See Bond et al. (2020) and De Ridder et al. (2022) for a detailed analysis of this source of bias.

Firm-Level Productivity Process

We assume firm-level demand shocks, A_{kit}, are fixed over time. It follows that the composite V_{kit} is driven only by productivity shocks.[37] Following Carvalho and Grassi (2019), we assume that firm-level productivity, Z_{ikt}, follows the discretized random growth process introduced by Córdoba (2008). Specifically, firm productivity in sector k evolves on an evenly spaced log grid, $\Phi_k = \{1, \varphi_k, \varphi_k^2, \ldots, \varphi_k^S\}$, where φ_k is greater than 1 and where S is an integer. Note $\varphi_k^n = \varphi_k \varphi_k^{n-1}$. A firm's productivity follows a Markov chain on this grid where the associate matrix $\{P_{n,m}^{(k)}\}_{n,m \in [|1,S|]}$ of transition probabilities is such that $P_{n,n}^{(k)} = 1 - a_k - c_k$, $P_{n,n-1}^{(k)} = a_k$, $P_{n+1,n}^{(k)} = c_k$, $P_{1,1}^{(k)} = a_k + b_k$, and $P_{S,S}^{(k)} = b_k + c_k$ with $1 > a_k, b_k, c_k > 0$ and $a_k + b_k + c_k = 1$. The stationary distribution of this process is Pareto with a tail index equal to $\delta_k = \log(\frac{a_k}{c_k})/\log(\varphi_k)$. The three parameters φ_k, δ_k, and c_k characterize this productivity process.

As shown in Córdoba (2008) and Carvalho and Grassi (2019), this process satisfies Gibrat's law for productivity such that, away from the boundaries, firms' productivity growth is independent of its current level. Additionally, this law generates a stationary Pareto distribution of productivity (Gabaix 1999). Note, however, that in our environment, this does not immediately imply firm size satisfies these properties, due to the finite number of firms within sectors and variable markups.

In Section 4.3, we discuss an exercise where we add aggregate TFP shocks chosen to target the volatility of annual changes in aggregate output.

Calibration Strategy

We now describe how we assign values to the model's parameters: the two demand elasticities ε and σ, the two macro parameters relative risk aversion η and Frisch labor-supply elasticity f, the number of firms N_k, the demand shifter A_k, and the productivity parameters φ_k, δ_k, and c_k for each of our 275 sectors. Table 2 summarizes parameter values while Table 1 and Figure 1 describes the model fit.

In terms of macro parameters, we set the relative risk aversion to 1 (log utility) and the Frisch labor-supply elasticity to 1, both of which are standard values in the business-cycle literature.

We assume that in all sectors firms compete à la Cournot. We set the within sector elasticity $\varepsilon = 5$.[38] We calibrate the between sector elasticity σ to target the slope of a regression in first-differences (over time) of the inverse sectoral markup on the HHI. In the data, the coefficient

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[37]Although our analytic results do not take a stand on the importance of productivity versus quality-shifter firm-level shocks, in the data we construct sectoral output by deflating nominal value-added by industry price indices. The latter typically do not take into account high-frequency changes in quality shifters. Therefore, for consistency, we abstract from shocks to quality shifters.

[38]In Appendix E we provide sensitivity analysis for alternative values of ε ranging between 4 and 7 while recalibrating the remaining model parameters. Our quantitative results on markup cyclicality are fairly stable, while the volatility of aggregate output is increasing in ε.

Table 1: Calibration Targets

Panel A: Economy Wide targets

Moment	Source	Data	Model
Slope of $\Delta\mu_{t,k}^{-1}$ on $\Delta HHI_{t,k}$	Table 5	-0.35	-0.35
Constant. of volat. on market share	Table A2	0.27	0.27

Panel B: Sectoral targets

Moment	Source	Data	Model
Number of firms N_k [*]	Baseline sample	1453	1453
Revenue share [*]	Baseline sample	0.16%	0.16%
HHI (See Fig 1) [*]	Baseline sample	0.115	0.115

NOTE: The rows with a [*] refers to 275 moments, one per five-digit sectors, we therefore report the average across the 275 sectors.

of this regression is -0.35 as reported in column (2) of the Table 5 discussed in Section 4.1. In the model, taking first-differences of equation (8) implies $\Delta\mu_{t,k}^{-1} = -\frac{\left(\frac{\varepsilon}{\sigma}-1\right)}{\varepsilon}\Delta HHI_{t,k}$. Given our choice of ε, matching a slope of -0.35 requires $\sigma = 1.8$. Own-cost pass-through rates $\tilde{\alpha}_{ki}$, defined in footnote 21, are shaped by the two demand elasticities. Our baseline choices of σ and ε imply an own-cost pass-through rate of 0.63 for large firms (those with a market share of 57% or higher), which lies within the confidence intervals in Amiti et al. (2019) for Belgian large firms.[39]

We now discuss how we assign parameter values that vary across our 275 sectors to match salients features of our data in the period 1994-2019. We set the number of firms per sector, N_k, to the average number of firms in sector k observed in our data. We calibrate the constant sector-level demand shifter, A_k, to target the average revenue share of each of our 275 sectors in the data. For each sector k, we choose the tail parameter of the stationary distribution, δ_k, to match the average HHI in the data. Figure 1 reports HHI in the data against the model counterpart. The fit is good, as revealed by the fact that all dots lie close to the 45-degree line.[40] The grid parameter φ_k determines the range of values that the HHI can take as we vary δ_k. We choose the lowest φ_k such that this range of values contains the value of the HHI in the data for

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[39]About 360 firm-year observations have a market share above 57%, representing approximatively the top $0,004\%$ of the market-share distribution. Our model implies pass-through rates that are on the high end of estimates in Amiti et al. (2019) for Belgian firms and on the low end of estimates in Berman et al. (2012) for French exporters, and is consistent with findings in both papers that pass-through rates are decreasing in firm size. For alternative values of ε reported in Appendix E, pass-through rates are lower (e.g. 0.55 for large firms when $\varepsilon = 7$) and hence closer to the point estimates in Amiti et al. (2019).

[40]Given a guess of δ_k, we draw 1,000 samples of N_k firm-level productivities from the Pareto distribution characterized by δ_k. For each of these samples, we solve for firm-level market shares and compute the implied HHI. We then calculate the median HHI across the 1,000 samples, and iterate over δ_k until we match the HHI for a given sector in the data. We repeat this procedure for each of the 275 sectors.

Table 2: Baseline Calibration

Panel A: Economy Wide parameters

Parameters	Value	Description
η	1	relative risk aversion
f	1	Frisch elasticity of labor supply
f_0	1	labor disutility parameter
ε	5	substitution across firms
σ	1.8	substitution across sectors

Panel B: Sectoral parameters

Parameters	Value	Description
S	70	number of productivity bins
φ_k	1.091	median firm-level pdty process
a_k, c_k	0.348, 0.250	median firm-level pdty process
A_k	0.0015	median sectoral preference shifter

this sector.[41] Finally, we set the remaining parameter of the productivity process, c_k, such that in each sector, the conditional volatility of market-share for a hypothetical infinitesimal firm is equal to the constant in the regression of market-share volatility on market-share estimated across all sectors in the data and reported in Table A2.[42]

In what follows, we use the calibrated model as a data-generating process to simulate firm-level, sector-level, and aggregate-level time series. We use the simulated sector-level and firm-level panels to run the corresponding regressions that we run on actual data. We also compute aggregate business-cycle statistics using the simulated aggregate time-series, which we then compare with counterparts in the data.

4 Model meets Data

In this section, we interpret measures of firm, sector, and aggregate markup changes in our data through the lenses of our framework.[43] We start by documenting the relation between markups and concentration measures, both within and across sectors. We additionally show that within-

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[41]We choose the number of productivity bins $S = 70$. Higher values of S have a minor impact on our results.

[42]Using the baseline sample, we compute market share growth rates for each period. We then compute for each firm, over time, the standard deviation of its market share growth rate. In column (2) of Table A2, we report the regression of the latter measure of market share volatility on average market share. Alternatively, we compute the standard deviation of the growth rate of market-share in the cross-section as explain in the note of Table A2. In both case, we obtain a precise estimated constant of 0.27 (0.00).

[43]We present baseline results in the main text and supplement our analysis with extensive robustness checks in Appendix D. In Appendix C, we report further robustness exercises addressing possible concerns regarding different markup measures and sample selection.
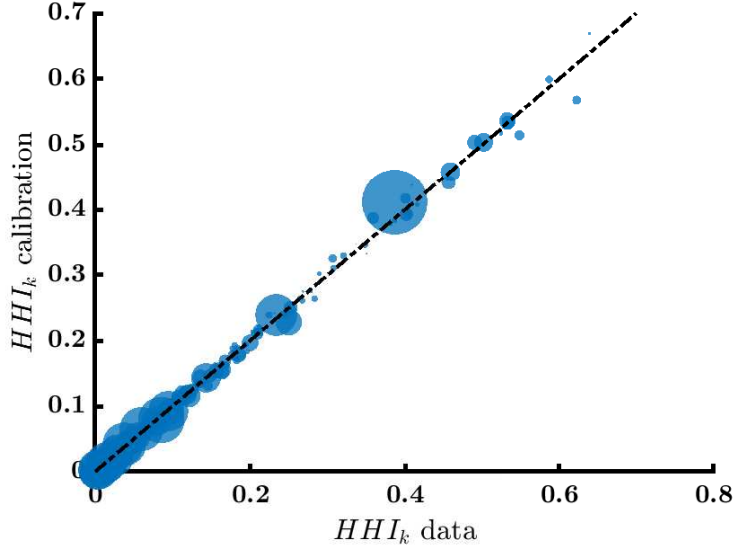
Figure 1: Model Fit

NOTE: For each sector, this figure show the HHI in the data (x-axis) against the median HHI computed over 1,000 samples drawn from the baseline calibration (y-axis). The size of each dot is proportional to the sector's average revenue share between 1994 and 2019.

firm markup dynamics account for a non-negligible fraction of sector-level markup changes. We then focus on unconditional reduced-form notions of markup cyclicality, showing that our model can rationalize the sign and magnitude of these alternative measures in the data. We conclude by quantifying the magnitude of aggregate markup movements in our granular setup and the role of endogenous markup.

4.1 Inspecting the Mechanism: Firm and Sector-level Evidence

4.1.1 Firm-level Evidence

Hardwired into our model is a key micro-level relationship between markups and concentration. At the firm-level, and following the discussion in Section 1.2, markups increase with a firm's market share. In turn, this immediately gives rise to a notion of markup pro-cyclicality at the micro-level: a firm's markup increases whenever its market share increases.

Taking the inverse of equation (5) and applying first differences yields a simple linear relation between the firm's market share and its inverse-markup,

$$\Delta \mu_{kit}^{-1} = -\frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon} \Delta s_{kit} \tag{35}$$

where $\Delta \mu_{kit}^{-1}$ is the first-difference of the inverse (gross) markup of firm i in sector k at time t and

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Δs_{kit} is the first-difference of its market share. This motivates the following simple empirical specification,

$$\Delta \mu_{kit}^{-1} = \alpha_t + \beta \Delta s_{kit} + \epsilon_{kit}, \tag{36}$$

where β is the coefficient of interest, which the model predicts to be negative. We allow for year fixed-effects α_t to control for unobserved markup shifters that are common across all firms. In alternative specifications we further allow for sector-year fixed effects, α_{kt}, thus absorbing markup variation that is common across all firms in a sector, in a given year. While these fixed effects are not present in our theoretical model they nevertheless allow us to empirically control for arbitrary aggregate and sector-specific markup trends in the data and serve as a robustness check.

We start by inspecting these firm-level relations in the French data. Recall from our discussion in the previous section that we have estimated firm-level markups for French firms over the period 1994-2019. Firm-level market shares are immediate to calculate in data by dividing firm-level revenue by the corresponding five-digit NAF sector revenue. Taking first-differences yields time series for $\Delta \mu_{kit}$ and Δs_{kit} for about 1 million firms over the period 1995-2019, where we lose the first year of observations due to the first-difference transformation.

Table 3: Inverse Markup and Market Share

	(1)	(2)	(3)
Dependent Variable:		$\Delta \mu_{kit}^{-1}$	
Δs_{kit}	-0.268	-0.268	-0.293
	(0.092)	(0.093)	(0.099)
Year FE	N	Y	N
Sector $\times$ Year FE	N	N	Y
Observations	8,051,767	8,051,767	8,051,767

NOTE: $\Delta \mu_{kit}^{-1}$ is the first difference of the inverse of firm i sector k gross markup between t and $t-1$, and Δs_{kit} gives the first difference of market share of firm i in sector k. Columns (1)-(3) report baseline empirical estimates for the FICUS-FARE (1995-2019) data. Column (1) reports pooled estimates while columns (2) and (3) report estimates that further control for year or sector×year fixed effects, respectively. Sector-year fixed effects are defined at the 5-digit NAF sector classification. Standard errors (in parentheses) are two-way clustered at the firm and year level. $\Delta \mu_{kit}^{-1}$ is winsorized at the 3% level.

Table 3 displays our estimates and the associated two-way (at firm and year level) cluster-robust standard errors. Column (1) displays the firm-level relation in first-differences, obtained by pooling all firm-level data (across sectors and years) for a total of over 8 million observations. This yields a negative and statistically significant coefficient, as theory predicts. Further, the empirical estimates remain stable and significant when additionally controlling for year (column 2) and sector-year (column 3) fixed effects. Finally, as a further robustness check, in Table A4 of Appendix D.1 we report estimates for an alternative specification that regresses firm

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markups on firm market shares in levels, allowing for firm fixed effects. The estimates are again similar to those in first-differences in columns (1) and (2) of Table 3.

The data is therefore consistent with changes in a firm's market share acting as a proximate driver of its markup dynamics, as predicted by theory. Notice however that, ultimately, in the model, a firm's market share and markup are jointly determined in equilibrium by exogenous firm-level technology (and/or quality) shifters. All else constant, a decrease in firm's marginal cost relative to that of its competitors will increase its competitiveness in the product market and, hence, its market share (and therefore its markup, as above). We now turn to assessing this relation in the data.

To do so, recall from Section 3.2 that, for the estimation sample, we can obtain both firm-level price data, P_{kit}, and markup estimates, μ_{kit}. Given this, for firms in this smaller estimation sample we can exploit the relation $P_{kit}/\mu_{kit} = mc_{kit}$, to back out an empirical proxy of firm-level marginal costs, mc_{kit}. We can thus inspect the model-implied predictions above regarding marginal costs, market shares and markups, albeit in a significantly smaller sample. Table 4 reports empirical estimates of simple OLS regressions of firm-level market share and markup growth rates on the growth rates of our firm-level marginal cost proxy.

Table 4: Markups, Market Shares and Marginal Costs

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent Variable:		$\Delta \log s_{kit}$			$\Delta \log \mu_{kit}$	
$\Delta \log mc_{it}$	-0.022	-0.022	-0.023	-0.091	-0.091	-0.096
	(0.003)	(0.003)	(0.003)	(0.008)	(0.009)	(0.009)
Year FE	N	Y	N	N	Y	N
Sector × Year FE	N	N	Y	N	N	Y
Observations	178,368	178,368	178,368	178,368	178,368	178,368

NOTE: $\Delta \log \mu_{kit}$ is the first-difference of (log) gross markup of firm i sector k at time t, $\Delta \log s_{kit}$ is the first-difference of (log) market share, and $\Delta \log mc_{it}$ is the first-difference of (log) marginal cost when the latter is defined as the difference between (log) price and (log) markup of firm i in sector k at time t. Columns (1)-(6) report empirical estimates for the estimation sample FARE (2009-2019) data. Columns (1) and (4) report pooled estimates while columns (2), (3), (5) and (6) report estimates that further control for year or sector×year fixed effects. Sector-year fixed effects are defined at the 5-digit NAF sector classification level. Standard errors (in parentheses) are two-way clustered at the firm and year level. Variables are winsorized at the 3% level.

Starting with the simple bivariate relation between marginal cost growth and market share growth, our estimates in column (1) – where we pool data across all sectors and years – imply that a one percent year-on-year increase in firm-level marginal costs translates to a two percent decrease of a firm's market share growth. This estimate is robust to additionally controlling for average economy-wide marginal cost dynamics (i.e. the year fixed effects specification in column 2) or the average marginal cost growth across competitors in a given firm's sector (i.e. the sector-year fixed effects specification in column 3). Our evidence is therefore consistent with

the main model mechanism. The second panel of Table 4 completes the argument by inspecting the relation between firm-level marginal cost growth and firm-level markups. Again, we observe that year-on-year increases in marginal costs results, as the model predicts, in lower markups, both unconditionally (in column 4) and when conditioning on year or sector-year fixed effects in columns (5) and (6), respectively. Finally, Tables A5 and A6 of Appendix D.1 we again verify that these empirical estimates are robust to considering an alternative specification in levels (rather than growth rates) with firm fixed-effects. Taken together, we conclude that the data is consistent with the basic qualitative firm-level predictions of our model.

4.1.2 Sector-level Evidence

As discussed in Section 1.3, equilibrium aggregation of firm-level outcomes yields additional predictions at the sector-level. First, note that, by the same logic as above, taking the inverse of equation (8) and then first-differences, yields the following relation between inverse sectoral markup and the sector's HHI:

$$\Delta \mu_{kt}^{-1} = -\frac{\frac{\varepsilon}{\sigma} - 1}{\varepsilon} \Delta HHI_{kt}, \tag{37}$$

where $\Delta \mu_{kt}^{-1}$ is the first-difference of the inverse (gross) markup of sector k at time t and ΔHHI_{kt} is the first-difference of its HHI. This yields a sector-level counterpart to equation (35) where now inverse sectoral markups comove linearly with sectoral concentration. To assess this relationship in the data, we consider the following empirical specification:

$$\Delta \mu_{kt}^{-1} = \alpha_t + \beta \Delta HHI_{kt} + \epsilon_{kt}, \tag{38}$$

where β is the coefficient of interest and where again we allow for year fixed effects α_t and additionally consider robustness to the inclusion of broad 2-digit sector-year fixed effects. Finally, in Appendix D.1, we additionally report estimates based on a levels specification and sector-level fixed effects.

To construct sectoral markups within 275 narrowly defined five-digit sectors, we aggregate firm-level markups to their sector-level counterparts by taking an harmonic market-share weighted average of firm-level markups - as indicated by the model equation (7).[44] For each these five-digit sectors, we construct the HHI by summing the squared firm-level market shares. For both the sector-level markup and HHI series, we then take first-differences across time periods. We obtain a balanced panel of 275 five-digit sectors across 25 years for a total of 6,875 observations.

Table 5, column (1) displays estimates of a pooled regression across all sectors and years, with

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[44]To compute the sector-level markup, we first winsorized the underlying firm-level inverse markup at the 3% level.

Table 5: Sector Inverse Markup and Sector HHI

	(1)	(2)	(3)
Dependent Variable:		$\Delta\mu_{kt}^{-1}$	
ΔHHI_{kt}	-0.354	-0.354	-0.350
	(0.172)	(0.172)	(0.166)
Year FE	N	Y	N
Sector(2D)×Year FE	N	N	Y
Number of Sectors	275	275	275
Observations	6,875	6,875	6,875

NOTE: $\Delta\mu_{kt}^{-1}$ is the first difference of sector k (inverse) markup in year t, ΔHHI_{kt} is the first difference of HHI in sector k. Columns (1)-(3) report empirical estimates for the FICUS-FARE (1995-2019) data, aggregated to the five-digit NAF sector level. Column (1) reports pooled estimates while columns (2) and (3) report estimates that further control for year or broad 2-digit-sector×year fixed effects, respectively. Standard errors (in parentheses) are two-way clustered at the sector and year level. Underlying firm-level inverse markups are winsorized at 3%.

and without year fixed-effects. Our estimates indicate a negative and significant relation between the change in concentration and the change in the inverse of sector markups. Further, our estimates remain stable when additionally considering year or broader 2-digit sector-year fixed effects in columns (2) and (3), respectively. Finally, note that estimates are similar if instead of growth rates we consider alternative specifications in levels, as shown in Table A7 in the Appendix D.1. In our model calibration, we target a slope of -0.35 in the specification (38), as reported in Table 1.

Note that our model additionally imposes cross-equation restrictions. Comparing equations (35) and (37), the slope coefficients of these two relations - that is, the slope of the inverse of firm markup on market share and the slope of the inverse sector markup on sector HHI - should coincide. Comparing point estimates across Tables 3 and 5, suggests that the implied slopes are indeed similar: focusing on the more demanding sector-year fixed effects specifications, we obtain slopes of -0.293 and -0.350, respectively, with both estimates falling within (less than) a one standard-error confidence interval from each other.

Finally, recall that in our model, changes in sectoral markups reflect two forces. First, for given firm market shares, the evolution of endogenous firm-level markups may lead to changes in aggregated, sector-level markups. Second, for given heterogeneous firm-level markups, equilibrium reallocation of market shares also impact sector markup dynamics. Specifically, note that following equation (7), the change in sectoral markups between two time periods can be written as

$$\Delta\mu_{kt}^{-1} = \sum_{i=1}^{N_k} \Delta\mu_{kit}^{-1}\,\overline{s}_{kit} + \sum_{i=1}^{N_k} \Delta s_{kit}\,\overline{\mu}_{kit}^{-1},$$

where Δ denotes the year-on-year difference and bars denote averages over two consecutive

years. This yields a standard within-between decomposition of sectoral markup changes. The first term on the right-hand-side gives the within term, measuring the change in (inverse) sectoral markup due to changes in firm-level markups, evaluated at the average market share of each firm. The second term on the right-hand-side is the between or reallocation term: it measures the change in the (inverse) sectoral markup due to the changes in firm market share, evaluated at a firm's average (inverse) markup. As discussed in Section 2.2 and shown in Appendix A.1, in the model under Cournot competition the within and between terms are equal to each other in every sector. From this result, it follows that, under Cournot, the contribution of the within and between/reallocation terms are each predicted to be equal to 50%.

Given time series of firm-level markups and market shares in the data, the within-between decomposition above can be readily computed. To do this, for each sector, we regress the within term over time on changes in sector-level markup. The coefficient of this regression is the contribution of the within term to the evolution of sector-level markups.[45] We find that for the median sector in the French data the within term accounts for 53% of the changes in sector markups, close to the model prediction 50%. While there is heterogeneity in the data, for half of the sectors in France, the contribution of the within term lies between 27% and 79%. Overall, taking firm and sector evidence together, the data is consistent with key predictions of the model.

4.2 Reduced-Form Varieties of Markup Cyclicality

Our theoretical framework yields a simple relation between markups and size: the level of a firm's markup is determined by its market share within a sector. In turn, both markups and market shares are driven by firm-level marginal cost shifters. The aggregation of within-firm endogenous markup changes and reallocation of market shares across firms determines sectoral markups and sectoral concentration, yielding a linear relation between a sector's (inverse) markup and its concentration. As we have seen, the data broadly supports these model-implied relations.

By contrast, a large applied literature investigates different definitions of "markup cyclicality." This literature yields a variety of results, with some contributions arguing for pro-cyclicality and others concluding in favor of counter-cyclicality.

In this section, we argue these conflicting empirical results can be largely ascribed to the alternative reduced-form exercises pursued and, in particular, to the reduced-form definitions of markup cyclicality being deployed in the literature. Importantly, as we show, our model with firm-level shocks only can go a long away in accounting for these seemingly conflicting

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[45]In the data, the reallocation term is not only due to changes in market share across continuing firms, but also due to churning as some firms enter and exit the market each year. We define the empirical reallocation term as the residual obtained from the difference between the change in (inverse) sectoral markup and the within term.

reduced-form relations in the data.

4.2.1 Firm-level Evidence

We start by analyzing a firm-level notion of reduced-form markup cyclicality and ask how do firm markups covary with the respective sector-level output.

Before going to the data, recall that our setting is a granular one in which extensive within-sector heterogeneity in the firm-size distribution enables large firm dynamics to lead the sector business cycle. In particular, in our setting with idiosyncratic firm-level shocks – and if pass-through rates do not fall too strongly with market shares – sector-output fluctuations are necessarily led by shocks to very large firms. To make matters concrete, consider a reduction in marginal cost for a large market-share firm. Given the granular nature of the economy, the corresponding sector output will typically increase (see equation 26). In addition, the large shocked firm will increase its market share and markup. This implies, as indicated in Proposition 4, that markups of large firms should comove positively with sector output.

By the same token, the average (small) firm in a given sector loses market share to the very largest firms: if sector-level output expansions are led by large firms, the latter will increase their market share whereas the average firm loses competitiveness - as evaluated by its market share within the sector. Again, due to the markup-market-share relation in our setting, this implies the average firm-markup is expected to comove negatively with sector output, as summarized by Proposition 4.

To evaluate this implication of the model, we implement the following reduced-form regression, both in the data and in our model-simulated data:

$$\log(\mu_{kit}) = \alpha_i + \gamma_t + \beta_1 \log Y_{k,t} + \beta_2 \log Y_{k,t} \times s_{kit} + \epsilon_{it}, \tag{39}$$

where $\log(\mu_{kit})$ is firm i sector k (log) gross markup in year t, Y_{kt} is sector k's (log) value-added in year t and s_{kit} is firm-level market share for firm i in sector k at year t.[46] Given the specification in log-levels, α_i is a firm fixed effect, which controls for time-invariant firm-level unobservables determining the average level of a firm's markup, while γ_t is a year fixed effect.[47] In this specification, β_1 captures the average correlation between firm markups and their respective sector output, and coefficient β_2, in the interaction term, captures heterogeneity in this relation as a function of a firm's market share.[48] For robustness, we additionally consider a specification in first differences of (log) firm markups, $\Delta\log(\mu_{kit})$, and (log) sectoral value added, ΔY_{kt} and

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[46]To obtain sector value-added, we sum firm-level nominal value-added to the NAF five-digit level and deflate using EU-KLEMS sectoral price deflators.

[47]We drop the year 1994 to have the same year of coverage as in section 4.1. For this reason, the number of observations is lower than what is reported in Table A1. Including the year 1994 gives very similar results.

[48]According to our model, given the parameters ε and σ, the market share suffices to determine the markup (equation 5). For this reason, we estimate equation (39) without further controls.

no firm-level fixed effects. In Appendix D.2 we consider additional robustness with alternative measure of sector output.

Table 6: Firm Markup and Sector Output

	(1) Data	(2) Data	(3) Model	(4) Model
Dependent variable:	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$
Y_{kt}	-0.073		-0.001	
	(0.008)			
$Y_{kt} * s_{kit}$	0.574		0.265	
	(0.044)			
ΔY_{kt}		-0.024		-0.001
		(0.009)		
$\Delta Y_{kt} * s_{kit}$		0.280		0.247
		(0.040)		
Firm FE	Y	N	Y	N
Year FE	Y	N	Y	N
Number of Observations	9,039,476	8,051,767	-	-

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and Y_{kt} sector k's (log) value-added in year t. $\Delta\log(\mu_{kit})$ is the first-difference of (log) gross markup in year t for firm i sector k, s_{kit} gives the market share of firm i in sector k, year t and ΔY_{kt} is the first-difference of sector k (log) value-added in year t. Columns (1) and (2) report empirical estimates for the FICUS-FARE (1995-2019) data. Standard errors are two-way clustered at the sector×year level. Columns (3) and (4) report estimates based on model-simulated data. Log and first-difference of log markup are winsorized at the 3% level.

Before proceeding, note that Hong (2017) considers a version of this regression, where Y is aggregate (rather than sector) value-added, using data for four large European countries. For these data, Hong (2017) estimates a negative β_1 and a positive β_2 estimate, concluding that (i) in the data "markups are countercyclical" and (ii) that there is "substantial heterogeneity in markup cyclicality across firms, with small firms having significantly more countercyclical markups than large firms."

Columns (1) and (2) of Table 6 summarize the estimates obtained when implementing the above reduced-form regression on our French firm-level data, in both levels (with firm fixed effects) and first differences of logged variables. The implied point estimates β_1 are negative and significant in both cases: the markup of the average firm is "countercyclical" with respect to own-sector output. Though the point estimates differ in magnitude across empirical specifications, the qualitative behavior of the average firm's markup is therefore consistent with the model intuition above. Further, we additionally confirm that there is substantial cross-sectional heterogeneity in this relation. In particular, the estimates on the interaction term – for either specification – imply that large firms, roughly with market shares above 10% (in the top 0.1% of the market share distribution), are procyclical with respect to the dynamics of sec-

toral output.[49]

Columns (3) and (4) of Table 6 implement the same reduced-form regressions on model-simulated data for 399,520 firms distributed across 275 five-digit sectors.[50] The model is able to reproduce the qualitative patterns observed in the data. Consistent with Proposition 4, markups for the average firm are countercyclical with respect to own-sector output, whereas large firm markups are procyclical. Furthermore, point estimates for the implied large firm procyclicality are of the same order of magnitude in both model-simulated and French data, and particularly close for the first-differences specification. Finally, in Appendix D.2 we show that these relations are robust to alternative definitions of sectoral output dynamics, by considering log-deviations of sector real value added from its trend (defined by either a Hodrick-Prescott or Hamilton (2018) filters) instead. These further checks again confirm that large firm markup dynamics are procyclical with respect to own sectoral output, whereas the average firm is not.

As discussed above, underlying the prediction of the model for heterogeneous cyclicality of firm-level markups is the fact that, in our granular environment, large firms' market shares are correlated positively with sector output whereas small firms' market shares are countercyclical. To assess this mechanism in the data we estimate the following regression:

$$\log(s_{kit}) = \alpha_i + \gamma_t + \beta \log Y_{k,t} + \epsilon_{kit},$$

where $\log(s_{kit})$ is the (log of) market share of firm i in sector k, $\log Y_{k,t}$ is the (log of) sector k real value-added, α_i is a firm-level fixed effect, and γ_t is a year fixed effect. Again, as above, we additionally consider a first difference specification (without firm fixed effects). In either specification, β measures the relation between market share and sector value-added. To assess the predicted heterogeneous behavior of firm-level market shares with respect to sector output, we implement this regression (i) on the whole sample of firms, (ii) on the subsample of firms whose average market share is lower than 50%, and (iii) on the subsample of firms whose average market share is above 50%.

Columns (1) and (4) in Table 7 report estimates of β on the full sample of the data and on the model-simulated data, respectively. As before, we experiment with both levels (and firm-fixed effects) and first-differences specifications. Both in the data and in the model – and for both specifications – the average firm's market share is counter-cyclical. Columns (2) and (5) report estimates for the subsample of firms whose market share is lower than 50%. Estimates of β are negative both in the data and in the model. Columns (3) and (6) report estimates for the subsample of firms whose market share is above 50%: consistently with our argument, estimates

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[49]As discussed above, Hong (2017) reports a version of this regression where aggregate value-added is interacted with an indicator for large firm. When we estimate a version of this reduced-form specification where sector value-added is interacted with an indicator for market-shares over 50%, we find a coefficient of 0.240 (0.044) on the interaction term for the log-level specification and of 0.026 (0.013) for the first-difference specification.

[50]The number of firms is smaller in the simulation than in the model as our model abstract from entry and exit and targets the yearly average number of firms in the economy over our period.

Table 7: Firm Market Share and Sector Output

	(1) Data (all data)	(2) Data ($\bar{s}_{ki} < 0.50$)	(3) Data ($\bar{s}_{ki} > 0.50$)	(4) Model (all data)	(5) Model ($\bar{s}_{ki} < 0.50$)	(6) Model ($\bar{s}_{ki} > 0.50$)
Dependent variable:			$\log s_{kit}$			
Y_{kt}	-0.594	-0.595	0.144	-2.613	-2.621	0.535
	(0.009)	(0.009)	(0.060)			
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Number of Obs.	9,039,476	9,039,036	440	-	-	-
Dependent variable:			$\Delta \log s_{kit}$			
ΔY_{kt}	-0.488	-0.488	0.091	-2.585	-2.591	0.274
	(0.018)	(0.018)	(0.037)			
Firm FE	N	N	N	N	N	N
Year FE	N	N	N	N	N	N
Number of Obs.	8,251,767	8,251,340	427	-	-	-

NOTE: s_{kit} gives the market share of firm i in sector k, year t, and Y_{kt} is the deviation of sector k (log) value-added in year t from its mean. $\Delta \log s_{kit}$ gives the first-difference of (log) market share of firm i in sector k, year t, and ΔY_{kt} is the first-difference of sector k (log) value-added in year t. $\bar{s}_{ki}$ is the average market share of firm i in market k. Column (1-3) reports empirical estimates for the FICUS-FARE (1995-2019) data. Sectors are defined at the 5-digit NAF sector classification level. First-difference in log market shares are winsorized at the 3% level. Standard errors in the data are two-way clustered at the sector×year level. Column (4-6) reports estimates based on model-simulated data.

of β are now positive, both in the data and in the model, although the magnitude is smaller in the data. Additionally, in Appendix D.2 we confirm that these findings are robust to alternative de-trending techniques for sectoral output dynamics.

Taken together, these results provide support for a key implication of our granular model with firm-level shocks. The average firm's market share and markup are countercyclical with respect to its own sector value-added, whereas large firms' market share and markups are procyclical.

4.2.2 Sector-Level Evidence

We now explore sector-level notions of markup cyclicality. We first ask how sector markups covary with own-sector output. Recall that in our granular setting, sectoral business cycles are driven by large firm dynamics, and that shocks to large firms induce a positive relationship between changes in sector-level output and markups. As encoded in Proposition 3, we should expect a positive correlation between sector markup and sector output.

To assess this relationship in the model and in the data, we follow Nekarda and Ramey (2013) and consider the following sector-level empirical specification:

$$\Delta \log \mu_{kt} = \alpha_k + \gamma_t + \beta \Delta Y_{kt} + \epsilon_{kt}, \tag{40}$$

where $\Delta \log \mu_{kt}$ denotes the first-difference of sector k's (log) markup, and ΔY_{kt} denotes the first difference of sector k's (log) real value-added. Sector-level markups are aggregated from firm-level estimates according to a harmonic weighted average, as indicated by the model equation (7).[51] We measure sector value-added in the data as in the previous section. We follow Nekarda and Ramey (2013) and include sector and year fixed effects to control for possibly heterogeneous trends in sector level variables. For robustness, we consider an alternative specification where we use sectoral variables in log deviations from trend (rather than first differences) and the trend is estimated following Hamilton (2018). In Appendix D.3 we report additional robustness, considering fixed-effect regressions for variables in levels and alternative detrending procedures.

Nekarda and Ramey (2013) estimate a positive and significant β in US sectoral data using a similar specification, concluding that "markups are generally procyclical (...) hitting troughs during recessions and reaching peaks in the middle of expansions."

Columns (1) and (2) in Table 8 report estimates of β in our French data for both first differences and trend-deviation specifications. Sector markups comove positively and significantly with sector output, which is consistent with the findings in Nekarda and Ramey (2013) despite

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[51] Nekarda and Ramey (2013) measure sectoral markup using various measure of the inverse of the labor share at the sectoral level. We construct sector-level markup from the aggregation of firm-level markup based on equation 34 with material as a variable input.

Table 8: Sector Markup and Sector Output

Dependent variable:	(1) Data $\Delta \log \mu_{kt}$	(2) Data $\widehat{\log}\mu_{kt}$	(3) Model $\Delta \log \mu_{kt}$	(4) Model $\widehat{\log}\mu_{kt}$
ΔY_{kt}	0.248		0.110	
	(0.053)		(0.040)	
$\widehat{Y}_{kt}$		0.174		0.117
		(0.050)		(0.035)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1 and 3), and Hamilton (2018) trend deviation of markup (columns 2 and 4), $\Delta \log \mu_{kt}$ and $\widehat{\log}\mu_{kt}$ respectively on sector value-added ΔY_{kt} and $\widehat{Y}_{kt}$ respectively. Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Sectors are defined at the 5-digit NAF sector classification level. Columns (3-4) report estimates based on model-simulated data. The point estimates for these columns give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

differences regarding the country of analysis, sample period, and the methods deployed to estimate markups. Appendix D.3 further confirms the robustness of this correlation to alternative empirical specifications and detrending procedures.

Columns (3) and (4) in Table 8 reports estimates of β in model-simulated data, applying the same empirical specifications as in the French data. We report the median and standard deviation of β estimates over 5,000 samples of 25 years each. The model implies a positive correlation between sector markup and sector output, yielding point estimates that are smaller but in the same order of magnitude as in the data.

To further understand the previous result, recall that sector-level markups in our model are linked to sector-level concentration as measured by the HHI, a relationship we explored empirically in Section 4.1.2. Therefore, underlying the cyclicality of sector-level markup is the cyclicality of the sectoral HHI. In our granular environment, in a typical sectoral expansion a few large firms expand by increasing their market share while smaller firms lose market share, resulting in higher concentration. To assess this mechanism in the data, we estimate a similar specification to equation 40, with sectoral HHI rather than sectoral markup on the left hand side:

$$\Delta \log HHI_{kt} = \alpha_k + \gamma_t + \beta \Delta Y_{kt} + \epsilon_{kt}.$$

Here, $\Delta \log HHI_{kt}$ is the first-difference in sector k's (log) HHI and ΔY_{kt} denotes the first-difference of sector k's (log) output. We include sector and year fixed effect as in the markup cyclical-

Table 9: Sector Concentration and Sector Output

Dependent variable:	(1) Data $\Delta \log HHI_{kt}$	(2) Data $\widehat{\log}HHI_{kt}$	(3) Model $\Delta \log HHI_{kt}$	(4) Model $\widehat{\log}HHI_{kt}$
ΔY_{kt}	0.332		0.533	
	(0.067)		(0.235)	
$\widehat{Y}_{kt}$		0.281		0.726
		(0.049)		(0.288)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1 and 3), and Hamilton (2018) trend deviation of HHI (columns 2 and 4) $\Delta \log HHI_{kt}$ and $\widehat{\log}HHI_{kt}$ respectively on sector value-added ΔY_{kt} and $\widehat{Y}_{kt}$ respectively. Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Sectors are defined at the 5-digit NAF sector classification level. Columns (3-4) report estimates based on model-simulated data. The point estimates for these columns give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

ity specification, equation (40). For robustness, we consider an alternative specification where we use log deviations from trend computed as in Hamilton (2018). In appendix D.3 we report additional robustness.

Columns (1) and (2) in Table 9 report estimates of β in our French data for both first differences and trend-deviation specifications. Sector concentration comoves positively and significantly with sector output. We next apply the same empirical specification in model simulated data, and we calculate the median and standard deviation of β estimates over 5,000 samples of 25 years each. Columns (3) and (4) in Table 9 show positive and significant estimates of β, consistent with the data. Table A11 in appendix D.3 confirms the robustness of this correlation to alternative empirical specifications and detrending procedures.

The more recent work by Bils et al. (2018) explores yet another reduced-form notion of markup cyclicality. Unlike the previous specification which relates changes in sectoral markups and changes in sectoral output, Bils et al. (2018) measure the cyclical comovement between sectoral markup and aggregate real GDP.

To understand this form of comovement in the context of our model, note that sector markups only react to within-sector firm shocks. Over long samples, under the conditions of Proposition 5, the model implies (i) positive comovement of a sector's markup with aggregate GDP and (ii) that this comovement is nevertheless weaker than that between a sector's markup and its output. Over a given cyclical episode - or more generally, in small samples - the model predic-

tion is ambiguous. A positive comovement is expected if the fluctuation in aggregate economic activity is driven by large firms in the same sector. However, aggregate output movements also reflect shocks hitting other sectors in the economy. If a sector comoves negatively with aggregate output, a negative correlation of that sector's markup with aggregate output will obtain. Overall, we should expect a weaker relationship between the average sector's markup and aggregate GDP fluctuations than between sectoral markups and sectoral output.

To explore this notion of cyclicality, we implement the following regression:

$$\Delta \log \mu_{kt} = \alpha_k + \beta \Delta Y_t + \epsilon_{kt}, \tag{41}$$

where $\Delta \log \mu_{kt}$ is the first-difference of sector k's log markup in year t, ΔY_t gives the first difference of (log) aggregate value-added in year t. α_k is a sector fixed effect that controls for sector-specific trends in markups. Sector-level markups are computed as above by taking a weighted harmonic mean of firm-level markups and aggregate value-added is computed by summing firm-level value-added deflated by the respective EU-KLEMS sector-level deflator. Finally, for robustness we again consider an alternative specification in log deviations from a Hamilton (2018) trend rather than growth rates. In appendix D.3, we consider another specification with different detrending.

Bils et al. (2018) consider a version of this regression based on US data. They conclude that "the price markup is estimated to be highly countercyclical" with the possible exception of service industries, for which they find evidence favoring procyclicality.[52]

Columns (1) and (2) of Table 10 summarize our estimates based on French data. While the point estimates are negative, they are also noisy: we do not find a statistically significant relation between sectoral markups and aggregate GDP. As shown in Table A12 of appendix D.3, this finding is robust to consider other detrending methods. For the average French sector, the data suggests that this reduced-form relation is not statistically significantly different from zero.

We now explore the connection between sector-specific markups and aggregate output implied by our model. Table 10 present median estimates (along with their respective standard deviations) of β over 5,000 samples of 25 year length. For the baseline calibration, reported in columns (3) and (4), our model implies a positive point estimate. However, the model simulations point to considerable uncertainty over this point estimate which is not statistically different from zero, as in the data.

As we will discuss in the next subsection, our model with only idiosyncratic productivity shocks understates aggregate output volatility. In order to match the observed volatility of aggregate output, we consider an extension with aggregate productivity shocks. Firm-level productivity is

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[52]Bils et al. (2018) measure markup using intermediate input share computed from sector level data from KLEMS. Instead, we construct sector-level markup by the aggregation of firm-level markup based on equation 7. Similarly, we use material as a variable input to estimate firm-level markup using equation 34.

given by $\widetilde{Z}_t \times Z_{ikt}$, where $\widetilde{Z}_t$ is normally distributed and independent across periods with volatility set to match the volatility of aggregate output in the data.[53] For this alternative parameterization, reported in columns (5) and (6), our model implies a point estimate roughly equal to zero. Of the 5,000 samples, about 20 to 30% (depending on how we filter the model-generated data) display sectoral markups that are countercyclical with respect to aggregate output. Intuitively, aggregate productivity shocks enhance aggregate output volatility but do not affect the relative firm-level productivity and therefore do not affect the market-share and markup distributions. It follows that aggregate shocks do not affect sector-level markups. Point estimates with aggregate shocks are therefore smaller than without aggregate productivity shocks and closer to its empirical counterpart.

We thus conclude that, consistent with the data, the model is able to generate a weak and noisy comovement between sectoral markups and aggregate output (in comparison to the stronger relation between sectoral markups and sectoral output).

4.3 Aggregate Markup Cyclicality and Output Fluctuations

In this final section, we turn our attention to fluctuations in aggregate markups and output. In the model, we first consider only idiosyncratic firm-level shocks according to the Markov process introduced above. Recall that in our environment, these shocks constitute the only source of markup and output fluctuations at the firm, sector, and aggregate levels. We then introduce aggregate productivity shocks to fully account for aggregate output volatility in our data.

Using our FICUS-FARE data, we construct aggregate markups, μ_t, as a weighted harmonic mean of firm-level markups, and aggregate GDP, Y_t, as the sum of firm-level value-added. We detrend these variables using the Hamilton (2018) filter. Using our calibrated model, we simulate 5,000 samples of 25-year firm-level histories. We implement the same procedure to construct detrended time-series of model simulated aggregate output and markup. For robustness, we also consider a first-differences specification. Table 11 presents data- and model-based estimates of the correlation and standard deviation of aggregate output and markups.

We first consider aggregate markup cyclicality. Recall from expression (33) that our model implies a positive comovement between aggregate output and aggregate markups, unless larger sectors have lower markups or, for finite samples, if a particular expansionary episode is driven by a sector with a sufficiently low markup, in which case negative comovement may obtain. That is, whereas we should observe positive comovement over sufficiently long samples, in any given short sample, comovement may be absent or negative depending on sectors driving the aggregate dynamics.

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[53]We set the standard deviation for $\log \widetilde{Z}_t$ to 2.20% (resp. 2.04%) for the specification in first-difference (resp. in deviation from trend)

Table 10: Sector Markup and Aggregate Output

Dependent variable:	(1) Data $\Delta \log \mu_{kt}$	(2) Data $\widehat{\log}\mu_{kt}$	(3) Model without Aggr. Shocks $\Delta \log \mu_{kt}$	(4) Model without Aggr. Shocks $\widehat{\log}\mu_{kt}$	(5) Model with Aggr. Shocks $\Delta \log \mu_{kt}$	(6) Model with Aggr. Shocks $\widehat{\log}\mu_{kt}$
ΔY_t	-0.327		0.165		0.008	
	(0.176)		(0.101)		(0.042)	
$\widehat{Y}_t$		-0.235		0.169		0.017
		(0.195)		(0.119)		(0.044)
Share negative coefficients	-	-	0.02	0.02	0.29	0.21
Sector FE	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector k's markup in year t in first-differences ($\Delta \log \mu_{kt}$, in columns 1 and 3) and Hamilton (2018) trend deviation ($\widehat{\log}\mu_{kt}$, in columns 2 and 4) on (log) aggregate real value-added in year t in either first-differences or Hamilton (2018) trend deviation (ΔY_t and $\widehat{Y}_t$, resp.). Columns (1) and (2) report empirical estimates for the FICUS-FARE (1995-2019) data. Regressions are weighted by average sectoral value-added. Standard errors (in parentheses) are clustered at the year level. Columns (3) and (4) report estimates based on model-simulated data. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples. Columns (5) and (6) report estimates based on model-simulated data with aggregate TFP shocks. Point estimates for this column are computed as for columns (3) and (4). The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output (either measured as a deviation from trend or a log first-difference) in France. The line "Share negative coefficients" gives the share of simulation with negative estimated coefficients in regression on the model-based simulations.

In Table 11, we can see that both in the data and in the model, the correlation between aggregate markup and aggregate output is positive. Our model predicts, however, much higher correlation than that observed in the data: the correlation is, at most, 0.13 in the data and 0.91 (computed as the median correlation across shock realizations) in the model.[54]

Next, we examine the magnitude of aggregate, granular fluctuations in output and markups implied by our model. Recall from our analytic results that incomplete pass-through weakens (relative to the specification of the model with heterogeneous but constant markups) the response of aggregate output to firm-level shocks, as implied by equation (31) — derived under parameter restrictions that we relax in our quantitative analysis.

Table 11 shows that, for the detrended specification, the standard deviation of aggregate output is 3.16% in our French data and 0.83% in our model (median across samples). That is, the volatility of aggregate output in our purely granular model is 26% of the volatility in the data. This result is robust to alternatively computing correlations on first-differences in the data and in model simulated samples. How large are granular movements in aggregate markups? The ratio of the standard deviation of aggregate markup to that of aggregate output is 59% in the data and 36% in our calibrated model (median across samples).

Although our model with firm-level shocks generates non-negligible fluctuations in aggregate output and markups, it only accounts for a fraction of aggregate fluctuations in the data. Moreover, as we discussed above, the correlation of markups and output is much higher than that in the data. In what follows, we show that if we superimpose on our calibrated model aggregate productivity shocks – in order to match aggregate output volatility in the data – the procyclicality of markups is much lower and closer to the data.

As discussed in the previous section, we assume firm-level productivity is given by $\widetilde{Z}_t \times Z_{ikt}$, where $\widetilde{Z}_t$ is normally distributed and independent across periods. We choose the standard deviation of $\widetilde{Z}_t$ to match the volatility of aggregate output in the data. Column (3) of Table 11 shows the implied business-cycle moments for the median 25-year sample. As discussed in Section 1, aggregate shocks do not affect firm market shares and markups, and hence the volatility of aggregate markups is unchanged relative to the model with only firm-level shocks. Because movements in output driven by aggregate productivity shocks are uncorrelated with markups, the correlation between aggregate markup and output falls to 0.27. In 16% of our 25-year samples, aggregate markups are countercyclical.[55]

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[54]Our model predicts large variation in the correlation coefficient and in the relative volatility of aggregate markups and output across small samples, depending on which sectors are driving aggregate dynamics and the relative levels of their markups. To see this variation at play, Figure A1 in Appendix D plots the histogram of correlation coefficients $\rho(\mu_t, Y_t)$ and relative standard deviations $\sigma(\mu_t)/\sigma(Y_t)$ across our 5,000, 25-year samples.

[55]We also considered second-moment shocks to firm-level productivity as in Bloom et al. (2018). An increase in the dispersion of firm-level productivity shocks reallocates market shares toward large firms, increasing the aggregate markup, but also raise aggregate output (Oi-Hartman-Abel effect). That is, in our model second moment shocks increase the correlation between aggregate markup and output.

Table 11: Aggregate Markup and Aggregate Output

	(1) Data			(2) Model without aggr. shocks			(3) Model with aggr. shocks		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\widehat{Y}_t$	3.16	1	1	0.83	1	1	3.16	1	1
$\widehat{\mu}_t$	1.87	0.59	0.01	0.30	0.36	0.91	0.30	0.09	0.29
ΔY_t	3.28	1	1	0.72	1	1	3.28	1	1
$\Delta \mu_t$	2.23	0.68	0.13	0.26	0.36	0.91	0.26	0.08	0.27

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for deviations from trend computed as in Hamilton (2018) of (log) aggregate output $\widehat{Y}_t$ and (log) aggregate markup $\widehat{\mu}_t$, and, for log first-difference of aggregate output ΔY_t and aggregate markup $\Delta \mu_t$. Column (1) reports empirical estimates for the FICUS-FARE (1995-2019) data. Column (2) reports the median over 5,000 simulated samples, each of 25 years. Column (3) reports the average over 5,000 simulated samples of 25 years from a model with aggregate TFP shocks. The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output (either measured as a deviation from trend or a log first-difference) in France.

Role of changes in firm-level markups for aggregate results Whereas the within/between decomposition in Section 4.1 demonstrates the importance of changes in firm-level markups to account for changes in aggregate markups in the model and data, as we discussed in Section 2, it is not necessarily informative of how different movements in aggregate output and markups would be if firm-level markups were heterogeneous but fixed over time.

To answer this question, we apply quantitatively the first-order-approximation analytic expressions in Section 2. In Appendix A.6, we provide expressions for correlations and volatilities under variable markups versus constant markups, given market shares and markups in the initial equilibrium. Because in our model, the distribution of firms by sector changes every period, we consider 1,000 independent samples drawn from the equilibrium in our quantitative model.[56]

Consider first movements in aggregate output. We compare the standard deviation of aggregate output under variable markups with that under heterogeneous but constant markups, given the same initial firm-level expenditure shares and markups and assuming the same volatility of firm-level shocks.[57] For the median sample, the standard deviation of aggregate output under

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[56]The magnitudes of correlations and ratios of standard deviations based on the first-order approximations are remarkably close to those in our quantitative non-linear results. For the median sample, the standard deviation of aggregate markup relative to output is 0.42 (vs. 0.36 in our quantitative non-linear results) and the correlation between aggregate markup and output is 0.87 (vs. 0.91 in our quantitative non-linear results).

[57]Market shares of large firms are less volatile under variable markups than under constant markups (see equation (A4) in the appendix). One could compare aggregate volatilities under these two specifications after adjusting

variable markups is 0.87 times that under heterogeneous but constant markups (the 95% confidence interval calculated across samples is 0.82-0.97). As explained in Section 2, incomplete pass-through (given pass-through rates that are decreasing in size) reduces aggregate output volatility in a similar way that a decline in firm concentration does.[58]

Consider now movements in aggregate markups. For the median sample, the standard deviation of aggregate markups under variable markups is 1.08 times that under heterogeneous but constant markups (the 95% confidence interval is 1.00-1.18). The median volatility of aggregate markups relative to output is 0.42 under variable markups and 0.34 under heterogeneous but constant markups. The median correlation between markups and output is 0.87 under variable markups and 0.92 under constant markups (the 95% confidence interval for the difference in correlations is [0-0.07]).

Overall, we see the magnitude and cyclicality of aggregate markups in our model are not too different when we counterfactually fix markups at their initial, heterogeneous level. Of course, rather than exogenously fixing markups, our model provides a unified theory of both markup (level) heterogeneity across firms and endogenous markup changes. Furthermore, this theory is consistent with a number of observations about markup changes in the data at the firm, sector, and aggregate levels.

5   Conclusion

In this paper, we examine markup cyclicality through the lens of a simple oligopolistic macroeconomic model with rich implications for the behavior of markups at the firm, sector, and aggregate levels. Working with administrative firm data for France, we show the model can reproduce qualitatively, and many times quantitatively, an array of markup-related empirical moments at various levels of disaggregation. Within our framework and measure of markups, we can reconcile seemingly conflicting variants of "markup-cyclicality" that have been considered in the literature. Finally, our granular oligopolistic setting produces non-negligible aggregate fluctuations, both in output and markups.

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the size of firm-level shocks to keep the same average volatility of market-shares (which, recall, is a target in our baseline calibration). If we match an unweighted average of these market-share volatilities, our results remain roughly unchanged. If we target a weighted average of these market-share volatilities, aggregate volatility is slightly higher under variable markups. In all cases, variable markups have a limited impact on reducing aggregate output volatility.

[58]Whereas variable markups reduce the volatility of aggregate output, markup heterogeneity per se contributes to higher aggregate volatility. By equation (30), markup heterogeneity under constant markups does not affect output changes with linear disutility of labor ($f \to \infty$). However, with finite labor disutility, reallocation of economic activity across heterogeneous markup firms is an additional source of output fluctuations, as studied in Baqae and Farhi (2019). In our model, the median standard deviation under heterogeneous and constant markups is 1.18 times that under homogeneous and constant markups (the 95% confidence interval is 1.13-1.22). Combining both results, the standard deviation under variable and heterogeneous markups is 1.02 times that under homogeneous and constant markups (the 95% confidence interval is 0.99-1.14). That is, output volatility under variable markups is only slightly higher than under constant and homogeneous markups.

One obvious route for future work is to superimpose in our model alternative shocks and frictions. Along this line, prime candidates would be to consider price setting and customer-accumulation frictions (see, e.g., Gilchrist et al. 2017 and Afrouzi 2019), as well as aggregate monetary and financial shocks. Relatedly, we have focused on static, intra-temporal markup decisions in which movements in markups are the result of changes in the shape of the demand curve in response to firm-level shocks. These forces would remain relevant even if one were to extend the model to allow for richer inter-temporal dynamics that result in more complex dynamic markup strategies (see e.g., Rotemberg and Saloner 1986). Bringing the resulting firm, sector, and aggregate dynamics to data - and comparing them against the forces in our static benchmark - would then render possible an assessment of the empirical merits of this more general approach.

Finally, extensions to more realistic and richer product and market structures would allow us to more accurately map model objects to the increasingly detailed micro data available to researchers. Such extensions would likely include multi-product firms, interlinked through intermediate-inputs, with some firms competing only locally in spatially segmented (product and factor) markets and others doing so nationwide and/or internationally.

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, "The Network Origins of Aggregate Fluctuations," *Econometrica*, 09 2012, *80* (5), 1977–2016.

Ackerberg, Daniel A., Kevin Caves, and Garth Frazer, "Identification Properties of Recent Production Function Estimators," *Econometrica*, 2015, *83* (6), 2411–2451.

Ackerberg, Daniel, C. Lanier Benkard, Steven Berry, and Ariel Pakes, "Econometric Tools for Analyzing Market Outcomes," in J.J. Heckman and E.E. Leamer, eds., *Handbook of Econometrics*, Vol. 6 of *Handbook of Econometrics*, Elsevier, January 2007, chapter 63.

Afrouzi, Hassan, "Firm Competition and the Cyclicality of Markups," 2019.

Aghion, Philippe, Antonin Bergeaud, Timo Boppart, Peter J. Klenow, and Huiyu Li, "Good Rents versus Bad Rents: R&D Misallocation and Growth," Working Paper August 2023.

Amiti, Mary, Oleg Itskhoki, and Jozef Konings, "International Shocks, Variable Markups, and Domestic Prices," *The Review of Economic Studies*, 02 2019, *86* (6), 2356–2402.

Anderson, Eric, Sergio Rebelo, and Arlene Wong, "Markups Across Space and Time," Working Paper 24434, National Bureau of Economic Research March 2018.

Arkolakis, Costas and Monica Morlacco, "Variable Demand Elasticity, Markups, and Pass-Through," Technical Report, Yale University 2017.

Atkeson, Andrew and Ariel Burstein, "Pricing-to-Market, Trade Costs, and International Relative Prices," *American Economic Review*, December 2008, *98* (5), 1998–2031.

Azar, José and Xavier Vives, "General Equilibrium Oligopoly and Ownership Structure," *Econometrica*, 2021, *89* (3), 999–1048.

Bao, Renjie, Jan De Loecker, and Jan Eeckhout, "Are Managers Paid for Market Power?," Working Paper 29918, National Bureau of Economic Research April 2022.

Baqaee, David and Ariel T Burstein, "Welfare and output with income effects and taste shocks," *NBER Working Paper*, 2021, (w28754).

___ **and Emmanuel Farhi**, "The Darwinian Returns to Scale," Working Paper 27139, National Bureau of Economic Research May 2020.

Baqaee, David Rezza and Emmanuel Farhi, "Productivity and Misallocation in General Equilibrium," *The Quarterly Journal of Economics*, 09 2019. qjz030.

Berger, David W., Kyle F. Herkenhoff, and Simon Mongey, "Labor Market Power," NBER Working Papers 25719, National Bureau of Economic Research, Inc March 2019.

Berman, Nicolas, Philippe Martin, and Thierry Mayer, " How do Different Exporters React to Exchange Rate Changes? *,"* *The Quarterly Journal of Economics*, 01 2012, *127* (1), 437–492.

Bilbiie, Florin O., Fabio Ghironi, and Marc J. Melitz, "Endogenous Entry, Product Variety, and Business Cycles," *Journal of Political Economy*, 2012, *120* (2), 304 – 345.

Bils, Mark, "The Cyclical Behavior of Marginal Cost and Price," *The American Economic Review*, 1987, *77* (5), 838–855.

___ **, Peter J. Klenow, and Benjamin A. Malin**, "Resurrecting the Role of the Product Market Wedge in Recessions," *American Economic Review*, April 2018, *108* (4-5), 1118–46.

Bloom, Nicholas, Max Floetotto, Nir Jaimovich, Itay Saporta-Eksten, and Stephen J. Terry, "Really Uncertain Business Cycles," *Econometrica*, 2018, *86* (3), 1031–1065.

Blundell, Richard and Stephen Bond, "GMM Estimation with Persistent Panel Data: an Application to Production Functions," 2000, *19* (3), 321–340.

Boar, Corina and Virgiliu Midrigan, "Markups and Inequality," Working Paper 25952, National Bureau of Economic Research June 2019.

Bond, Steve, Arshia Hashemi, Greg Kaplan, and Piotr Zoch, "Some Unpleasant Markup Arithmetic: Production Function Elasticities and their Estimation from Production Data," Working Paper 27002, National Bureau of Economic Research April 2020.

Brooks, Wyatt J, Joseph P Kaboski, and Yao Amber Li, "Growth Policy, Agglomeration, and (the Lack of) Competition," Working Paper 22947, National Bureau of Economic Research December 2016.

Burstein, Ariel and Gita Gopinath, *International Prices and Exchange Rates*, Vol. 4, Elsevier,

Carvalho, Vasco M. and Basile Grassi, "Large Firm Dynamics and the Business Cycle," *American Economic Review*, April 2019, *109* (4), 1375–1425.

___ **and Xavier Gabaix**, "The Great Diversification and Its Undoing," *American Economic Review*, August 2013, *103* (5), 1697–1727.

Córdoba, Juan Carlos, "A Generalized Gibrat's Law," *International Economic Review*, November 2008, *49* (4), 1463–1468.

De Loecker, Jan and Frederic Warzynski, "Markups and Firm-Level Export Status," *American Economic Review*, May 2012, *102* (6), 2437–71.

＿ **and Jan Eeckhout**, "Global Market Power," Technical Report 2018.

＿ **, ＿ , and Simon Mongey**, "Quantifying the Effect of Market Power: An Oligopolistic Framework for Firm Dynamics," Technical Report 2018.

De Ridder, Maarten, Basile Grassi, and Giovanni Morzenti, "The Hitchhiker's Guide to Markup Estimation," techreport 17532, CEPR 2022.

Deb, Shubhdeep, Jan Eeckhout, Aseem Patel, and Lawrence Warren, "Market Power and Wage Inequality," Working Papers 1360, Barcelona School of Economics September 2022.

di Giovanni, Julian and Andrei A. Levchenko, "Country Size, International Trade, and Aggregate Fluctuations in Granular Economies," *Journal of Political Economy*, 2012, *120* (6), 1083 – 1132.

＿ **, ＿ , and Isabelle Mejean**, "Firms, Destinations, and Aggregate Fluctuations," *Econometrica*, 07 2014, *82* (4), 1303–1340.

＿ **, ＿ , and ＿** , "The Micro Origins of International Business-Cycle Comovement," *American Economic Review*, January 2018, *108* (1), 82–108.

Doraszelski, Ulrich and Jordi Jaumandreu, "Using Cost Minimization to Estimate Markups," 2019.

D'Acunto, Francesco, Ryan Liu, Carolin Pflueger, and Michael Weber, "Flexible prices and leverage," *Journal of Financial Economics*, 2018, *129* (1), 46–68.

Edmond, Chris, Virgiliu Midrigan, and Daniel Yi Xu, "Competition, Markups, and the Gains from International Trade," *American Economic Review*, October 2015, *105* (10), 3183–3221.

＿ **, ＿ , and ＿** , "How Costly Are Markups?," Working Paper 24800, National Bureau of Economic Research July 2018.

Gabaix, Xavier, "Zipf'S Law For Cities: An Explanation," *The Quarterly Journal of Economics*, August 1999, *114* (3), 739–767.

＿ , "The Granular Origins of Aggregate Fluctuations," *Econometrica*, 05 2011, *79* (3), 733–772.

Gali, Jordi, "Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand," *Journal of Economic Theory*, June 1994, *63* (1), 73–96.

Gaubert, Cecile and Oleg Itskhoki, "Granular Comparative Advantage," Working Paper 24807, National Bureau of Economic Research July 2018.

Gilchrist, Simon, Raphael Schoenle, Jae Sim, and Egon Zakrajšek, "Inflation Dynamics during the Financial Crisis," *American Economic Review*, March 2017, *107* (3), 785–823.

Goldberg, Pinelopi Koujianou and Rebecca Hellerstein, "How rigid are producer prices?," Staff Reports 407, Federal Reserve Bank of New York 2009.

Gopinath, Gita, Oleg Itskhoki, and Roberto Rigobon, "Currency Choice and Exchange Rate Pass-Through," *American Economic Review*, March 2010, *100* (1), 304–36.

Grassi, Basile, "IO in I-O: Size, Industrial Organization, and the Input-Output NetworkMake a Firm Structurally Important," Working Papers 619, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University 2018.

Hall, Robert E., "The Relation between Price and Marginal Cost in U.S. Industry," *Journal of Political Economy*, 1988, *96* (5), 921–947.

Hamilton, James D., "Why You Should Never Use the Hodrick-Prescott Filter," *The Review of Economics and Statistics*, December 2018, *100* (5), 831–843.

Hong, Sungki, "Customer Capital, Markup Cyclicality, and Amplification," Working Papers 2017-33, Federal Reserve Bank of St. Louis April 2017.

Hottman, Colin J., Stephen J. Redding, and David E. Weinstein, "Quantifying the Sources of Firm Heterogeneity," *The Quarterly Journal of Economics*, 2016, *131* (3), 1291–1364.

Jaimovich, Nir and Max Floetotto, "Firm dynamics, markup variations, and the business cycle," *Journal of Monetary Economics*, 2008, *55* (7), 1238 – 1252.

Kikkawa, Ayumu Ken, Glenn Magerman, and Emmanuel Dhyne, "Imperfect Competition in Firm-to-Firm Trade," 2019.

Kimball, Miles S., "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit and Banking*, 1995, *27* (4), 1241–1277.

Levinsohn, James and Amil Petrin, "Estimating Production Functions Using Inputs to Control for Unobservables," *Review of Economic Studies*, 2003, *70* (2), 317–341.

Loecker, Jan De, Jan Eeckhout, and Gabriel Unger, "The Rise of Market Power and the Macroeconomic Implications*," *The Quarterly Journal of Economics*, 01 2020, *135* (2), 561–644.

⎯ , Pinelopi K. Goldberg, Amit K. Khandelwal, and Nina Pavcnik, "Prices, Markups, and Trade Reform," *Econometrica*, 2016, *84* (2), 445–510.

Luttmer, Erzo G.J., "Models of Growth and Firm Heterogeneity," *Annual Review of Economics*, 09 2010, *2* (1), 547–576.

Mongey, Simon, "Market Structure and Monetary Non-neutrality," Staff Report 558, Federal Reserve Bank of Minneapolis October 2017.

Morlacco, Monica, "Market Power in Input Markets: Theory and Evidence from French Manufacturing," Technical Report 2019.

Nekarda, Christopher J. and Valerie A. Ramey, "The Cyclical Behavior of the Price-Cost Markup," NBER Working Papers 19099, National Bureau of Economic Research, Inc June 2013.

⎯ and ⎯ , "The Cyclical Behavior of the Price-Cost Markup," *Journal of Money, Credit and Banking*, 2020, *52* (S2), 319–353.

Olley, G Steven and Ariel Pakes, "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, November 1996, *64* (6), 1263–97.

Pasten, Ernesto, Raphael Schoenle, and Michael Weber, "Price Rigidity and the Granular Origins of Aggregate Fluctuations," Working Papers Central Bank of Chile 864, Central Bank of Chile January 2020.

Rossi-Hansberg, Esteban, Pierre-Daniel Sarte, and Nicholas Trachter, "Diverging Trends in National and Local Concentration," in "NBER Macroeconomics Annual 2020, volume 35" NBER Chapters, National Bureau of Economic Research, Inc, 2020.

Rotemberg, Julio J and Garth Saloner, "A supergame-theoretic model of price wars during booms," *The American economic review*, 1986, *76* (3), 390–407.

Rotemberg, Julio J. and Michael Woodford, "The cyclical behavior of prices and costs," in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics*, Vol. 1 of *Handbook of Macroeconomics*, Elsevier, August 1999, chapter 16, pp. 1051–1135.

Smith, Dominic A. and Sergio Ocampo, "The Evolution of U.S. Retail Concentration," 2023.

Stroebel, Johannes and Joseph Vavra, "House Prices, Local Demand, and Retail Prices," *Journal of Political Economy*, 2019, *127* (3), 1391–1436.

Wang, Olivier and Iván Werning, "Dynamic oligopoly and price stickiness," *American Economic Review*, 2022, *112* (8), 2815–49.

____ **and Iván Werning**, "Dynamic Oligopoly and Price Stickiness," Working Paper 27536, National Bureau of Economic Research July 2020.

Appendix to "Bottom-up Markup Fluctuations"

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A   Theory Appendix

A.1   Global between / within decomposition of changes in sectoral markups

The change in the inverse of the sectoral markup between two time periods is, by equation (7),

$$\frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \left(\frac{s_{kit'}}{\mu_{kit'}} - \frac{s_{kit}}{\mu_{kit}} \right)$$

This change in sectoral markups can be decomposed into a within term (i.e., changes in firm-level markups evaluated at firms' expenditure share averaged over both time periods) and a between term (i.e., changes in expenditure shares evaluated at firm-level markups averaged over both time periods) as follows:

$$\frac{1}{\mu_{kt'}} - \frac{1}{\mu_{kt}} = \sum_{i=1}^{N_k} \frac{1}{2} \left[(s_{kit'} + s_{kit}) \left(\frac{1}{\mu_{kit'}} - \frac{1}{\mu_{kit}} \right) + \left(\frac{1}{\mu_{kit'}} + \frac{1}{\mu_{kit}} \right) (s_{kit'} - s_{kit}) \right] \qquad \text{(A1)}$$

Note that if markups are equal across firms (as is the case with $\sigma = \varepsilon$), then all terms in (A1) are equal to zero.

It is straightforward to show that, by equation (5) under Cournot competition, the within and the between terms in (A1) are equal to

$$\frac{1}{2} \sum_{i=1}^{N_k} (s_{kit'} - s_{kit}) (s_{kit'} + s_{kit}) \left(\frac{1}{\sigma} + \frac{1}{\varepsilon} \right).$$

Therefore, under Cournot competition, the contribution in changes in sectoral markups of the between and the within terms is 50% each, irrespective of the values of σ and ε (as long as $\sigma \neq \varepsilon$). If σ is close to ε, firm-level markups are less responsive to shocks (reducing the within term), but firm-level markups are also less heterogeneous across firms (reducing the between term).

With Bertrand competition, the within/between decomposition is not pinned down at 50-50.

A.2 Firm-level market shares

Combining $\widehat{s}_{kit} = \widehat{A}_{kit} + (1 - \varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt}\right)$, (16), and (18),

$$\widehat{s}_{kit} = \alpha_{ki}\left[\widehat{V}_{kit} - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}\widehat{V}_{ki't}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]. \tag{A2}$$

The response of firm i's expenditure share to a firm i shock is

$$\widehat{s}_{kit} = \alpha_{ki}\left[1 - \frac{s_{ki}\alpha_{ki}}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\widehat{V}_{kit}. \tag{A3}$$

Finally, we can express the variance of expenditure shares as

$$\mathbb{V}ar\left[\widehat{s}_{kit}\right] = \left(\frac{\alpha_{ki}\sigma_v}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right)^2\left[\left(\sum_{i'\neq i}^{N_k} s_{ki'}\alpha_{ki'}\right)^2 + \sum_{i'\neq i}^{N_k}\left(s_{ki'}\alpha_{ki'}\right)^2\right]. \tag{A4}$$

A.3 Changes in sectoral markups

Substituting equations (15), (16), (18), and

$$\widehat{s}_{kit} = \widehat{A}_{kit} + (1 - \varepsilon)\left(\widehat{P}_{kit} - \widehat{P}_{kt}\right), \tag{A5}$$

into equation (20) yields

$$\widehat{\mu}_{kt} = \mu_k \sum_{i=1}^{N_k} s_{ki}\alpha_{ki}\left[\left(\frac{\Gamma_{ki} - 1}{\mu_{ki}}\right) - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}\left(\frac{\Gamma_{ki'} - 1}{\mu_{ki'}}\right)}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]\widehat{V}_{kit}. \tag{A6}$$

Setting $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$, we obtain the expression for changes in sectoral markups under constant markups, (22).

Under our assumptions on market structure,

$$\frac{\Gamma_{ki} - 1}{\mu_{ki}} = \frac{\varepsilon - 1}{\varepsilon} - \frac{2}{\mu_{ki}}. \tag{A7}$$

$(\Gamma_{ki} - 1)/\mu_{ki}$ is increasing in markup μ_{ki} and hence also in market share s_{ki}. Substituting equation (A7) into (A6), we obtain expression (21).

Under Bertrand competition, markup elasticities Γ_{ki} are given by

$$\Gamma_{ki} \equiv \frac{\partial \log \mu_{ki}}{\partial \log s_{ki}} = \left[\varepsilon\left(\frac{\mu_{ki} - 1}{\mu_{ki}}\right) - 1\right](\mu_{ki} - 1),$$

and $(\Gamma_{ki} - 1)/\mu_{ki}$ by

$$\frac{\Gamma_{ki} - 1}{\mu_{ki}} = \varepsilon \left(\frac{\mu_{ki} - 1}{\mu_{ki}} \right)^2. \tag{A8}$$

Both Γ_{ki} and $\frac{\Gamma_{ki}-1}{\mu_{ki}}$ are increasing in markups and market shares. Changes in sectoral markups under Bertrand competition are

$$\widehat{\mu}_{kt} = \mu_k \varepsilon \sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \left[(\varepsilon - s_{ki} (\varepsilon - \sigma))^{-2} - \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'} (\varepsilon - s_{ki'} (\varepsilon - \sigma))^{-2}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} \right] \widehat{V}_{kit}.$$

As under Cournot, a positive shock to firm i results in an increase in sectoral markup if and only if firm i is sufficiently large in its sector.

To compare analytically changes in sectoral markups under constant markups (equation 15) and variable markups (equation 21), we restrict the extent of *ex-ante firm heterogeneity*. Specifically, we assume that sector k contains N_k^A type A firms and $N_k^B = N_k - N_k^A$ type B firms, and in the initial equilibrium, firms within each type have equal demand/productivity composite, V_{kit}. In the initial equilibrium, each firm of type $g = A, B$ has market share s_k^g, markup μ_k^g, and markup elasticity Γ_k^g. Firms of type A are indexed by $i = 1, ..., N_k^A$ and firms of type B are indexed by $N_k^A + 1, ..., N_k$. In this case, equation (A6) under Cournot competition can be written as

$$\widehat{\mu}_{kt} = \frac{2}{1 + (\varepsilon - 1)\widetilde{\Gamma}_k} \left[s_k^A \left(1 - \frac{\mu_k}{\mu_k^A} \right) \sum_{i=1}^{N_k^A} \widehat{V}_{kit} + s_k^B \left(1 - \frac{\mu_k}{\mu_k^B} \right) \sum_{i=N_k^A+1}^{N_k} \widehat{V}_{kit} \right], \tag{A9}$$

where

$$\widetilde{\Gamma}_k = N_k^B s_k^B \Gamma_k^A + N_k^A s_k^A \Gamma_k^B.$$

The term in square brackets in equation (A9) corresponds to the change in the sectoral markup under fixed markups as expressed above. Therefore, given the same firm-level shocks, sectoral markups change by more (and the variance is higher) under variable markups than under constant markups if and only if the term in front of the square brackets in equation (A9) is higher than 1, which is the case if $(\varepsilon - 1)\widetilde{\Gamma}_k < 1$. This condition is violated if σ is sufficiently low and/or ε sufficiently high.

Proof of Proposition 2 Define $f(s)$ and $g(s)$ as probability density functions defined over market shares in sector k, $s = s_{k1}, ..., s_{kN_k}$, given by $f(s) = \frac{s\alpha(s)}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}}$ and $g(s) = sf(s)a$ with $a = \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}}{\sum_{i'=1}^{N_k} s_{ki'}^2 \alpha_{ki'}} > 1$ and $\alpha(s)$ is defined in equation (17). Because the likelihood ratio $g(s)/f(s) = sa$ is increasing in s, $g(.)$ first-order stochastically dominates $f(.)$. If $s_{ki}\alpha_{ki}$ is increasing in s_{ki}, $f(s)$ is increasing in s. It then follows that $\sum_{i=1}^{N_k} [g(s_{ki}) - f(s_{ki})] f(s_{ki}) > 0$, which corresponds to inequality (25). Note that if $s_{ki}\alpha_{ki}$ is decreasing in s_{ki}, inequality (25) is reversed.□

Under what conditions is $s_{ki}\alpha_{ki}$ increasing in market shares, as required by Proposition 2? Un-

der Cournot competition,

$$s_{ki}\alpha_{ki} = \frac{\left(1 - \frac{1}{\varepsilon}\right)s_{ki} - \left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)s_{ki}^2}{1 - \frac{1}{\varepsilon} + (\varepsilon - 2)\left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)s_{ki}},$$

which is increasing in s_{ki} if and only if

$$2\left(\frac{\varepsilon - 1}{\varepsilon}\right)s_{ki} + \left(\frac{1}{\sigma} - \frac{1}{\varepsilon}\right)(\varepsilon - 2)s_{ki}^2 < \frac{\sigma(\varepsilon - 1)^2}{\varepsilon(\varepsilon - \sigma)}. \tag{A10}$$

Because the left-hand side of this equation is increasing in s_{ik} (for $s_{ik} \leq 1$), this inequality holds for $s_{ki} \leq \tilde{s}_k$, where $\tilde{s}_k$ is a function of σ and ε. This implies inequality (25) is satisfied if all market shares in sector k are less than or equal to $s_{ki} \leq \tilde{s}_k$. Note the condition that $s_{ki}\alpha_{ki}$ is increasing in s_{ki} is sufficient but not necessary for inequality (25) to hold. In particular, inequality (25) may hold (so that sectoral markups and prices comove negatively) even if $s_{ki}\alpha_{ki}$ is increasing in some range of the distribution of market shares in a sector but decreasing at the upper tail of the distribution.

A.4   Changes in sectoral productivity

By equations (9) and (15), changes in sectoral productivity are, up to a first order, given by

$$\widehat{Z}_{kt} = \sum_{i=1}^{N_k} s_{ki}\left[\left(\frac{\varepsilon}{\varepsilon - 1} - \frac{\mu_k}{\mu_{ki}}\right)\widehat{V}_{kit} - \varepsilon\left(1 - \frac{\mu_k}{\mu_{ki}}\right)\Gamma_{ki}\widehat{s}_{kit}\right],$$

where changes in market shares are given by (A5). The term $s_{ki} \times \frac{\varepsilon}{\varepsilon - 1}$ corresponds to the elasticity of sectoral productivity under monopolistic competition. The remaining terms reflect changes in efficiency due to reallocation across firms with heterogeneous markups, as discussed in detail in Baqaee and Farhi (2019).

A.5   Changes in sectoral and aggregate output

We now derive equation (30). We first calculate changes in aggregate output. Using equations (29) and (30), changes in aggregate output can be expressed in terms of changes in sectoral markup and price as

$$\widehat{Y}_t = (1 + f\eta)^{-1}\sum_k s_k\left[-\left(f + 1 + (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right)\right)\widehat{P}_{kt} + \frac{s_k\mu}{\mu_k}\widehat{\mu}_{kt}\right] \tag{A11}$$

In response to sector k shocks only, changes in aggregate output are

$$\widehat{Y}_t = (1 + f\eta)^{-1}s_k\left[-\left(f + 1 + (\sigma - 1)\left(1 - \frac{\mu}{\mu_k}\right)\right)\widehat{P}_{kt} + \frac{s_k\mu}{\mu_k}\widehat{\mu}_{kt}\right] \tag{A12}$$

and change in aggregate price by $\widehat{P}_t = s_k \widehat{P}_{kt}$.

Changes in sectoral output are given by

$$\widehat{Y}_{kt} = -\sigma \widehat{P}_{kt} + \sigma \widehat{P}_t + \widehat{Y}_t. \tag{A13}$$

In response to sector k shocks only, substituting changes in aggregate output and price using the expressions above, changes in sectoral output are given by equation (26).

Finally, expression (27) is obtained as follows. First, changes in firm-level markups are, combining equations (A2) and (15),

$$\widehat{\mu}_{kit} = \Gamma_{ki} \alpha_{ki} \left[\widehat{V}_{kit} - \frac{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'} \widehat{V}_{ki't}}{\sum_{i'=1}^{N_k} s_{ki'} \alpha_{ki'}} \right]. \tag{A14}$$

Changes in sectoral output when $f \to \infty$ are, by equations (26) and (18),

$$\widehat{Y}_{kt} = - \left[\sigma \left(1 - s_k \right) + \eta^{-1} s_k \right] \widehat{P}_{kt} = \frac{\left[\sigma \left(1 - s_k \right) + \eta^{-1} s_k \right]}{\varepsilon - 1} \frac{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki} \widehat{V}_{kit}}{\sum_{i=1}^{N_k} s_{ki} \alpha_{ki}}. \tag{A15}$$

Calculating $\mathbb{C}ov \left[\widehat{Y}_{kt}, \widehat{\mu}_{kit} \right]$ in the presence of shocks to all firms (only those in sector k are relevant), we obtain expression (27).

A.6 Volatility and covariance of aggregate markups and output

In this section, we provide expressions for the variance of and covariance between aggregate markups and aggregate output. We do not impose $f \to \infty$, as we do in the main text. We use these expressions in section 4.3.

The covariance between sector prices and markups, $\mathbb{C}ov \left[\widehat{\mu}_{kt}, \widehat{P}_{kt} \right]$, is given by (24) under variable markups and (23) under constant markups.

The variance of sectoral prices is

$$\mathbb{V}ar \left[\widehat{P}_{kt} \right] = \frac{\sigma_v^2}{(\varepsilon - 1)^2} \sum_{i=1}^{N_k} \left(\frac{\alpha_{ki} s_{ki}}{\sum_{i'} \alpha_{ki'} s_{ki'}} \right)^2. \tag{A16}$$

Under constant markups, $\Gamma_{ki} = 0$ and $\alpha_{ki} = 1$. The variance of the aggregate price is

$$\mathbb{V}ar \left[\widehat{P}_t \right] = \sum_k s_k^2 \mathbb{V}ar \left[\widehat{P}_{kt} \right]. \tag{A17}$$

A5

The variance of sectoral markups is

$$\mathbb{V}ar\left[\widehat{\mu}_{kt}\right] = \mu_k^2 \sum_{i=1}^{N_k} s_{ki}^2 \alpha_{ki}^2 \left[\left(\frac{\Gamma_{ki}-1}{\mu_{ki}}\right) - \frac{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}\left(\frac{\Gamma_{ki'}-1}{\mu_{ki'}}\right)}{\sum_{i'=1}^{N_k} s_{ki'}\alpha_{ki'}}\right]^2 \sigma_v^2. \tag{A18}$$

The variance of aggregate markups is

$$\mathbb{V}ar\left[\widehat{\mu}_t\right] = \sum_k s_k^2 \left[\left(\frac{\mu}{\mu_k}\right)^2 \mathbb{V}ar\left[\widehat{\mu}_{kt}\right] + (1-\sigma)^2\left(1-\frac{\mu}{\mu_k}\right)^2 \mathbb{V}ar\left[\widehat{P}_{kt}\right] - (\sigma-1)\left(1-\frac{\mu}{\mu_k}\right)\mathbb{C}ov\left[\widehat{\mu}_{kt},\widehat{P}_{kt}\right]\right] \tag{A19}$$

The covariance between aggregate price and markup is

$$\mathbb{C}ov\left[\widehat{P}_t,\widehat{\mu}_t\right] = \mu \sum_k \frac{s_k^2}{\mu_k}\mathbb{C}ov\left[\widehat{P}_{kt},\widehat{\mu}_{kt}\right] - (\sigma-1)\sum_k s_k^2\left(1-\frac{\mu}{\mu_k}\right)\mathbb{V}ar\left[\widehat{P}_{kt}\right]. \tag{A20}$$

The variance of aggregate output is

$$\mathbb{V}ar\left[\widehat{Y}_t\right] = \left(\frac{1}{1+\eta f}\right)^2 \mathbb{V}ar\left[\widehat{\mu}_t\right] + \left(\frac{1+f}{1+\eta f}\right)^2 \mathbb{V}ar\left[\widehat{P}_t\right] - \frac{(1+f)}{(1+\eta f)^2}\mathbb{C}ov\left[\widehat{P}_t,\widehat{\mu}_t\right] \tag{A21}$$

Finally, the covariance between aggregate output and markups is

$$\mathbb{C}ov\left[\widehat{Y}_t,\widehat{\mu}_t\right] = \left(\frac{1}{1+\eta f}\right)\mathbb{V}ar\left[\widehat{\mu}_t\right] - \left(\frac{1+f}{1+\eta f}\right)\mathbb{C}ov\left[\widehat{P}_t,\widehat{\mu}_t\right]. \tag{A22}$$

A.7 Decreasing returns to scale

The production function is now given by

$$Y_{kit} = Z_{kit}L_{kit}^{\beta}. \tag{A23}$$

where $\beta \leq 1$. Marginal cost is

$$MC_{kit} = \beta^{-1}W_t\left(Y_{kit}\right)^{(1-\beta)/\beta}\left(Z_{kit}\right)^{-1/\beta}, \tag{A24}$$

or, using $P_{kit}Y_{kit} = s_{kit}P_{kt}Y_{kt}$,

$$MC_{kit} = \beta^{-1}W_t\mu_{kit}^{\beta-1}\left(P_{kt}Y_{kt}s_{kit}\right)^{(1-\beta)}\left(Z_{kit}\right)^{-1}. \tag{A25}$$

The firm-level markup, μ_{kit}, is defined as the ratio of price to marginal cost, and is related to expenditure shares by equation (5), which does not depend on β.

Labor payments of firm i in sector k are

$$L_{kit}W_t = \beta\mu_{kit}^{-1}P_{kit}Y_{kit},$$

and profits (revenues minus labor payments) are

$$\Pi_{kit} = \left(1 - \beta\mu_{kit}^{-1}\right)P_{kit}Y_{kit}.$$

We define the sectoral markup as the ratio of sectoral revenues to labor payments,

$$\mu_{kt} \equiv \frac{P_{kt}Y_{kt}}{W_tL_{kt}}, \tag{A26}$$

which can be expressed as a function of firm-level markups and expenditure shares,

$$\mu_{kt}^{-1} = \beta\sum_{i=1}^{N_k}\mu_{kit}^{-1}s_{kit}. \tag{A27}$$

The 50-50 between/within decomposition of changes in sectoral markups under Cournot competition derived in the appendix holds irrespectively of the value of β.

The expenditure share of firm i in sector k, using $P_{kit} = \mu_{kit}MC_{kit}$, satisfies

$$s_{kit} = \frac{V_{kit}\left(\mu_{kit}^{\beta}s_{kit}^{1-\beta}\right)^{1-\varepsilon}}{\sum_{i'=1}^{N_k}V_{ki't}\left(\mu_{ki't}^{\beta}s_{ki't}^{1-\beta}\right)^{1-\varepsilon}}. \tag{A28}$$

Equilibrium firm-level expenditure shares and markups are the solution to equations (5) and (A28).

Log-linearizing (A28) and using $\widehat{\mu}_{kit} = \Gamma_{ki}\widehat{s}_{kit}$, we obtain the analog to equation (14):

$$\widehat{s}_{kit} = \widehat{V}_{kit} + (1-\varepsilon)\Lambda_{ki}\widehat{s}_{kit} - \sum_{i'=1}^{N_k}s_{ki'}\left(\widehat{V}_{ki't} + (1-\varepsilon)\Lambda_{ki'}\widehat{s}_{ki't}\right), \tag{A29}$$

where $\Lambda_{ki} = \beta\Gamma_{ki} + 1 - \beta$. Note that $\Gamma_{ki} < \Lambda_{ki}$ if and only if $\Gamma_{ki} < 1$.

We can follow similar steps to obtain expressions for changes in sectoral markups and prices to firm-level shocks, as well as the implied variances and covariances.

A.8 Markups when firms internalize impact on aggregates

In our baseline model we assume that when a firm chooses quantity, it does not take into account that its choice impacts aggregate output and the wage. This is a behavioral assumption since, with a discrete number of sectors and a discrete number of firms by sector, a firm's choice

does has a non-zero effect on aggregates. Here we solve for markups relaxing this assumption.

The inverse demand for firm i in sector k (omitting time subscripts) is

$$p_i \equiv \frac{P_{ki}}{P} = Y_{ki}^{-\frac{1}{\varepsilon}} \left(Y_k\right)^{\frac{1}{\varepsilon}-\frac{1}{\sigma}} Y^{\frac{1}{\sigma}}.$$

Differentiating p_{ki} with respect to Y_{ki}, taking other firms' quantities as given (but not sectoral or aggregate output), we obtain

$$\frac{d\log p_{ki}}{d\log Y_{ki}} = -\frac{1}{\varepsilon}\left(1 - s_{ki}\right) - \frac{1}{\sigma}s_{ki} + \frac{1}{\sigma}s_k s_{ki} \tag{A30}$$

where we used

$$\frac{d\log Y_k}{d\log Y_{ki}} = \frac{P_{ki}Y_{ki}}{P_k Y_k} = s_{ki}$$

and

$$\frac{d\log Y}{d\log Y_k} = \frac{P_k Y_k}{PY} = s_k.$$

The last term in (A30) is zero if we calculate this derivative taking Y as given. From labor supply choice, we can express the real wage as

$$w \equiv \frac{W}{P} = f_0 Y^{\eta} L^{\frac{1}{f}}$$

Differentiating w with respect to Y_{ki} and using the fact that $d\log Y_{ki} = d\log L_{ki}$ (since $Y_{ki} = Z_{ki}L_{ki}$), we obtain

$$\frac{d\log w}{d\log Y_{ki}} = \eta s_k s_{ki} + \frac{1}{f}s_k^n s_{ki}^n, \tag{A31}$$

where $s_k^n = \sum_{i \in k} L_{ki}/L$ and $s_{ki}^n = L_{ki}/L_k$.

Firm i chooses output Y_{ki} to maximize real profits (i.e. profits relative to the aggregate price index), $Y_{ki} \times \left[p_{ki}\left(Y_{ki}, Y_{-ki}\right) - \frac{w}{Z_{ki}}\left(Y_{ki}, Y_{-ki}\right)\right]$, taking output choices by other firms, Y_{-ki}, as given. We do not take into account the effect that changes in profits have on consumption and leisure of the firm's owner (Azar and Vives, 2021). The first order condition is

$$p_{ki} - \frac{w}{Z_{ki}} + Y_{ki}\left(\frac{dp_{ki}}{dY_{ki}} - \frac{dw}{dY_{ki}}\frac{1}{Z_{ki}}\right) = 0$$

which can be re-arranged as

$$p_{ki} = \frac{w}{Z_{ki}}\left(\frac{1 + \frac{d\log w}{d\log Y_{ki}}}{1 + \frac{d\log p_{ki}}{d\log Y_{ki}}}\right).$$

Substituting the expressions for $\frac{d\log p_{ki}}{d\log Y_{ki}}$ and $\frac{d\log w}{d\log Y_{ki}}$, (A30) and (A31), we obtain

$$P_{ki} = \frac{W}{Z_{ki}}\left(\frac{1 + \eta s_k s_{ki} + \frac{1}{f}s_k^n s_{ki}^n}{1 - \frac{1}{\varepsilon}\left(1 - s_{ki}\right) - \frac{1}{\sigma}s_{ki} + \frac{1}{\sigma}s_k s_{ki}}\right) \tag{A32}$$

Since markups now depend on economy-wide sales and employment shares, $s_k s_{ki}$ and $s_k^n s_{ki}^n$, rather than on the shares within sectors, we must solve for markups in all sectors simultaneously rather than sector by sector in our baseline model, which is more intensive computationally.

If $s_k \to 0$, then (A32) becomes

$$P_{ki} = \frac{W}{Z_{ki}} \left(\frac{1}{1 - \frac{1}{\varepsilon}(1 - s_{ki}) - \frac{1}{\sigma}s_{ki}} \right).$$
(A33)

This is the expression for prices in the baseline model, in which we assumed that $\frac{d\log w}{d\log Y_{ki}} = \frac{d\log Y}{d\log Y_{ki}} = 0$ when firms choose quantity.

Markups in expressions (A32) and (A33) differ for two reasons. First, a unilateral increase in Y_{ki}, raises Y, implying a smaller decline in price p_{ki} compared to the case in which individual firms take Y as given. This implies a higher effective demand elasticity, lowering markups. This effect is captured by the term $\frac{1}{\sigma}s_k s_{ki}$ in the denominator of (A32).

Second, an increase in Y_{ki} raises w. This reduces the profit maximizing quantity compared to the case in which w is taken as given by individual firms. This effect is smaller the higher is the Frish elasticity f and the less sensitive is the marginal utility of consumption to aggregate output. This effect is zero if labor disutility is independent of aggregate labor (e.g. for perfectly elastic labor supply $f = \infty$) and if the marginal utility of consumption is independent of aggregate consumption (e.g. for linear utility in consumption $\eta = 0$).

Applying (A32) using sales and employment shares in our baseline calibration has a negligible impact on markup levels compared to those based on (A33) our baseline. For example, across the two alternatives, the implied level of markups levels for the highest markup firms (where the effect would be largest) differ only at the third decimal place.

B Data, Estimation, and Calibration Appendix

B.1 Data appendix

In table A1, we report descriptive statistics for the estimation sample used in the estimation of the production function and for the baseline sample used to compute markups in our empirical exercise.

Table A1: Descriptive Statistics

Panel A: Estimation Sample

	#obs	mean	median	25th per.	75th per.	95th per.
Sales (log)	220,733	8.14	7.97	6.98	9.08	11.04
Quantity (log)	220,733	7.53	7.45	6.29	8.76	10.94
Price (log)	220,733	0.61	0.28	-.005	.90	2.95
Wage bill (log)	220,733	6.83	6.67	5.82	7.61	9.41
Capital (log)	220,733	6.92	6.74	5.59	8.08	10.37
Services (log)	220,733	6.72	6.61	5.55	7.73	9.60
Materials (log)	220,733	6.94	6.83	5.62	8.15	10.26

Panel B: Baseline Sample

	#obs	mean	median	25th per.	75th per.	95th per.
Sales (log)	9,383,228	5.85	5.58	4.69	6.77	8.86
Wage bill (log)	9,383,228	4.32	4.32	3.06	5.49	7.21
Capital (log)	9,383,228	4.00	3.84	2.93	4.92	7.09
Services (log)	9,383,228	4.20	3.90	3.01	5.13	7.26
Materials (log)	9,383,228	4.69	4.44	3.40	5.78	8.22
Markup	9,383,228	1.39	1.21	0.82	1.77	3.30
Elast. Materials	9,383,228	0.45	0.40	0.25	0.59	1.02
Sales/Materials	9,383,228	3.63	2.92	2.09	4.35	8.95
Local RTS	9,383,228	0.95	0.96	0.84	1.09	1.26

	#obs	mean	median	top 1%	top 0.1%	top 0.01%
Market Share (pp)	9,383,228	0.07	0.003	0.93	9.12	38.40

NOTE: Panel A (estimation sample) gives statistics for the sample of firms in the EAP survey from 2009 to 2019 with price and quantity information. Data is winsorized by two-digit sectors at 1%. Panel B (baseline sample) gives statistics of for the sample of firms in FICUS-FARE from 1994 to 2019 as described in the main text. Local Return to Scale (Local RTS) is defined as the sun of the material, labor, capital and service elasticities. Markup, material elasticity, ratio of sales to materials and local RTS are winsorized at the 3% level.

B.2 Markup Estimation

In this appendix, we describe the empirical framework we use to estimate production functions and firm-level markups. We also discuss its implementation in the FICUS-FARE French firm census data. This framework is based on the so-called production approach and builds on to the methodology in De Loecker and Warzynski (2012) and De Loecker et al. (2016, 2020) as discussed and implemented in De Ridder et al. (2022).

We assume all firms within two-digits sectors have a common production function, up to a firm-specific Hicks-neutral TFP. We further assume that this firm-specific TFP Z_{it} follows an AR(1) process in logs, that is, $\log Z_{it} = \rho \log Z_{it-1} + \xi_{it}$. For simplicity, in what follows, we omit sector notation. The production function of firm i is

$$Y_{it} = Z_{it} F\left(L_{it}, K_{it}, M_{it}, O_{it}\right), \tag{A34}$$

where Z_{it} denotes TFP, L_{it} denotes labor, K_{it} denotes capital, M_{it} denotes materials, and O_{it} denotes services. These inputs are homogenous across firms within sectors and traded in competitive markets. In our estimation of markups, we do not impose that F is constant returns to scale.

B.2.1 Recovering Markups

When minimizing costs, we assume that material is a variable input that is not subject to any adjustment cost or any intertemporal decision. Under these assumptions, the first-order-condition of the firms' cost-minimization problem for materials M_{it} can be rewritten as

$$P_t^M = \lambda_{it} Z_{it} \frac{\partial F}{\partial M} \quad \Leftrightarrow \quad \mu_{it} = \frac{P_{it} Y_{it}}{P_t^M M_{it}} \frac{Z_{it} \frac{\partial F}{\partial M}}{Y_{it}/M_{it}},$$

where $\lambda_{it} = \frac{P_{it}}{\mu_{it}}$; that is output price is equal to the markup over marginal cost. We denote by $\theta_{it}^M = \frac{Z_{it} \frac{\partial F}{\partial M}}{Y_{it}/M_{it}}$ the elasticity of the production function with respect to material input M_{it}. Markup is equal to the product of the ratio of sales to materials and the elasticity of the production function with respect to materials:

$$\mu_{it} = \frac{P_{it} Y_{it}}{P_t^M M_{it}} \theta_{it}^M. \tag{A35}$$

We calculate the ratio of sales to materials using the FICUS-FARE data (Panel B Table A1) on sales and input expenditures, and we estimate the production function as discussed in the next section.

B.2.2 Production-function estimation

In this subsection, we describe the production-function estimation procedure. We implement a two-stage procedure using a control-function approach, as introduced by Ackerberg et al. (2007, 2015), but adapted to an oligopolistic competition environment following De Ridder et al. (2022).

We implement the estimation procedure described below at the two-digit sector level. Given our assumptions that inputs are homogeneous and that firms are price takers in the input markets, we deflate input expenditures by sector-level price indices to recover inputs' quantities.

Below, we denote with small capital letters the logarithm of large capital letters: $z_{it} = \log Z_{it}$, $p_{it} = \log P_{it}$, $l_{it} = \log L_{it}$, $k_{it} = \log K_{it}$, $m_{it} = \log M_{it}$ and $o_{it} = \log O_{it}$. We assume that firms, in a given sector, produce by combining their inputs using a translog production function:

$$y_{it} = z_{it} + \sum_{u \in \{l,k,m,o\}} \beta_u u_{it} + \sum_{\{u,v\} \in \{l,k,m,o\}} \sum \beta_{uv} u_{it} v_{it} = z_{it} + X'_{it}\beta$$

where, in the last equality, we collect all the terms in the vector of data $X'_{it} = (l_{it}, k_{it}, m_{it}, o_{it}, l_{it}^2, k_{it}^2, m_{it}^2, o_{it}^2, l_{it}k_{it}, l_{it}m_{it}, l_{it}o_{it}, k_{it}m_{it}, k_{it}x_{it}, , m_{it}x_{it})$ and the vector of parameters to be estimated $\beta' = (\beta_l, \beta_k, \beta_m, \beta_o, \beta_{l^2}, \beta_{k^2}, \beta_{m^2}, \beta_{o^2}, \beta_{lk}, \beta_{lm}, \beta_{lo}, \beta_{km}, \beta_{ko}, \beta_{mo})$. Finally, we assume that quantity is observed with some measurement errors ϵ_{it}, that is, observed quantity $\widetilde{y}_{it}$ differs from actual quantity y_{it} such that

$$\widetilde{y}_{it} = y_{it} + \epsilon_{it} = X'_{it}\beta + z_{it} + \epsilon_{it}.$$

The estimation consists of two stages. First, we purge the observed quantity from the measurement errors ϵ_{it}. Second, we construct a dynamic panel GMM estimator to estimate the vector of parameters β.

The empirical counterpart of each variable is discussed in section 3.1 and descriptive statistics are given in Panel A of Table A1. The summary statistics of the data used in the estimation of markups can be found in Panel A of Table A1.

First-Stage. The first stage of this procedure consists of separating the measurement errors from the true quantity using the fact that firms observe their productivity z_{it} when deciding the amount of inputs. The demand for the variable input, here material m_{it}, can be expressed as a function of productivity: $m_{it} = m(z_{it}, \Xi_{it})$, where Ξ_{it} is a vector of all variables that determine m_{it} other than productivity. This function is often called the control function introduced by Olley and Pakes (1996) later extended in Levinsohn and Petrin (2003) and Ackerberg et al. (2015). Under the assumption that m_{it} rises monotonically in z_{it}, the demand function can be inverted,

such that $z_{it} = m^{-1}(m_{it}, \Xi_{it})$. Substituting this function in the production function gives

$$\widetilde{y}_{it} = y_{it} + \epsilon_{it} = X'_{it}\beta + m^{-1}(m_{it}, \Xi_{it}) + \epsilon_{it}.$$

The fitted values of a non-parametric regression of $\widetilde{y}_{it}$ on the variables in X'_{it}, m_{it} and Ξ_{it} therefore identify ϵ_{it}, as long as the the variables in Ξ_{it} that determine the demand for m_{it} are correctly specified.

To construct this control function, we use the first-order-condition with respect to the static input materials in the cost-minimization problem (as in equation A35)

$$P_t^M = \frac{P_{it}}{\mu_{it}} Z_{it} \frac{\partial F}{\partial M}. \tag{A36}$$

Using the fact that $\frac{\partial F}{\partial M}$ is a function of the inputs' usage, $\frac{\partial F}{\partial M}(L_{it}, K_{it}, M_{it}, X_{it})$, equation (A36) implicitly defines M_{it} as a function of productivity, Z_{it}, conditional on other inputs' usage L_{it}, K_{it}, X_{it}, material price, output price p_{it}, and markup. Furthermore, following De Ridder et al. (2022), we assume the markup is a function of market share, $\mu_{it} = \mu_t(s_{it})$, as is the case under the nested CES demand system in our model under either Cournot or Bertrand competition. In the data, this market share is defined as the ratio between firm-level sales and the sum of the sales of all firms in the same NAF sector, where market shares are defined at the five-digit level of sectoral disaggregation.

Equipped with this control function, we run non-parametric regression of $\widetilde{y}_{it}$ on the inputs usage and their interaction in X_{it}, market share s_{it} to control for markups, output price p_{it}, and a time-fixed effect to control for input price.[A59] The fitted values of this regression identified the measurement errors ϵ_{it} and allowed to recover the true quantity y_{it}.

Second-Stage. In the second stage, as in Ackerberg et al. (2015), we build a dynamic panel estimator similar in the spirit of Blundell and Bond (2000) where the identification is achieved through an instrument. Specifically, we used past values of input usage as instruments for current values. Following De Ridder et al. (2022), the GMM-based asymptotic estimator we study is defined as follows:

Definition 1 *The GMM estimator is $\widehat{\beta} \in \mathbb{R}^{14}$ and $\widehat{\rho} \in \mathbb{R}$ such that the moments $\mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]$ and $\mathbb{E}\left[\widehat{z}_{it-1}\widehat{\xi}_{it}\right]$ are equal to zero where $\widehat{z}_{it} = y_{it} - X'_{it}\widehat{\beta} = X'_{it}(\beta - \widehat{\beta}) + z_{it}$ and $\widehat{\xi}_{it} = \widehat{z}_{it} - \widetilde{\rho}\widehat{z}_{it-1} = (X_{it} - \rho X_{it-1})'(\beta - \widehat{\beta}) + X'_{it-1}(\beta - \widehat{\beta})(\rho - \widehat{\rho}) + z_{it-1}(\rho - \widehat{\rho}) + \xi_{it}$*

In the remainder of this appendix, we study the condition under which the above estimator admits solutions. To this end, let us study the following system of equations, which defined the

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[A59]In practice such non-parametric regression is performed by regressing the observed quantity on a third-order polynomial of the variables.

estimator and whose unknowns are $\widehat{\beta}$ and $\widehat{\rho}$:

$$\begin{cases} \mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right] = 0 \\ \mathbb{E}\left[\widehat{z}_{it-1}\widehat{\xi}_{it}\right] = \mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right]'(\beta - \widehat{\beta}) + \mathbb{E}\left[z_{it-1}\widehat{\xi}_{it}\right] = 0 \end{cases} \iff \begin{cases} \mathbb{E}\left[X_{it-1}\widehat{\xi}_{it}\right] = 0 \\ \mathbb{E}\left[z_{it-1}\widehat{\xi}_{it}\right] = 0 \end{cases} \iff$$

$$\begin{cases} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[X_{it-1}X'_{it-1}\right](\beta - \widehat{\beta})(\rho - \widehat{\rho}) + \mathbb{E}\left[X_{it-1}z_{it-1}\right](\rho - \widehat{\rho}) = 0 \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[z_{it-1}X'_{it-1}\right](\beta - \widehat{\beta})(\rho - \widehat{\rho}) + \mathbb{E}\left[z^2_{it-1}\right](\rho - \widehat{\rho}) = 0 \end{cases},$$

where we use $\mathbb{E}\left[X_{it-1}\xi_{it}\right] = 0$ and $\mathbb{E}\left[z_{it-1}\xi_{it}\right] = 0$, and, where we denote $\widetilde{X}_{it} = X_{it} - \rho X_{it-1}$. Note that the first line of the above system of equations corresponds to 14 equations, while the second line is just a scalar equation. We have $14 + 1$ equations with unknown $(\widehat{\beta}', \widehat{\rho}) \in \mathbb{R}^{14+1}$. In general, this system of equations has multiple solutions. However, when $(\widehat{\beta}', \widehat{\rho})$ is not too far from the true value (β', ρ), the terms in $(\beta - \widehat{\beta})(\rho - \widehat{\rho})$ are of second order. Ignoring these terms leads to the following reduced system which can be written in matrix form:

$$\begin{cases} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[X_{it-1}z_{it-1}\right](\rho - \widehat{\rho}) = 0 \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right](\beta - \widehat{\beta}) + \mathbb{E}\left[z^2_{it-1}\right](\rho - \widehat{\rho}) = 0 \end{cases} \iff \begin{pmatrix} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[X_{it-1}z_{it-1}\right] \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[z^2_{it-1}\right] \end{pmatrix} \begin{pmatrix} \beta - \widehat{\beta} \\ \rho - \widehat{\rho} \end{pmatrix} = 0$$

which admits a unique solution $(\widehat{\beta}, \widehat{\rho}) = (\beta, \rho)$ as long as the (15×15) matrix $\begin{pmatrix} \mathbb{E}\left[X_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[X_{it-1}z_{it-1}\right] \\ \mathbb{E}\left[z_{it-1}\widetilde{X}'_{it}\right] & \mathbb{E}\left[z^2_{it-1}\right], \end{pmatrix}$ is invertible. We conclude that the GMM estimator is locally identified and consistent.

B.3 Calibration Appendix

Table A2: Market Share and Market Share Volatility

Dependent Variable	$\sigma^{\overline{g_s}^i}_{kit}$	$\sigma^{\overline{g_s}^t}_{ki}$
Coefficient	(1)	(2)
s_{kit}	-0.536	
	(0.001)	
$\overline{s}_{ki}$		-0.839
		(0.026)
Constant	0.274	0.271
	(0.000)	(0.000)
Observations	9,358,228	833,285

NOTE: $\sigma^{\overline{g_s}^i}_{kit}$ is, for a firm i in sector k at time t, the standard deviation of the growth rate of market share across firm in the same market share percentile. $\sigma^{\overline{g_s}^t}_{ki}$ is the standard deviation of the growth rate of market share of firm i in sector k across time. Column (1) gives the regression of $\sigma^{\overline{g_s}^i}_{kit}$ on market share of firm i at time t, s_{kit}. Column (2) gives the regression of $\sigma^{\overline{g_s}^t}_{ki}$ on average market share of firm i across time, $\overline{s}_{ki}$.

C   Robustness Empirical Results

In this appendix, we discuss further robustness exercises. We consider the following variations of our baseline choices: using accounting (Lerner index) markups (section C.1), restricting the sample to the period covered by price data (section C.2), alternative outlier treatment (section C.3), computing markups using revenue data only (section C.4), estimating production functions for single product firms (section C.5), restricting the sample to the estimation sample (section C.6) and focusing on manufacturing firms (section C.7).

The empirical results for these robustness exercises are collected in Table A3, which display around 96 estimated coefficients. For convenience, column (1) displays the baseline results.

C.1   Accounting markup (Lerner Index)

Our baseline estimates and several of our robustness checks are based on the production function approach to recover markups, as described in section 3.2 and appendix B.2. In this robustness exercise, we instead compute markups using the "accounting approach" which consists of measuring accounting profits which, under constant return to scale and suitably normalized, will be equal to firm-level markup.

Specifically, we compute the Lerner index of firm i at time t as $Lerner_{it} \equiv \frac{P_{it}Y_{it} - TC_{it}}{P_{it}Y_{it}}$ where TC_{it} is the total cost measured as the sum of labor, capital, material and service expenditures, and $P_{it}Y_{it}$ is the total revenue of the firm i at time t.[A60] Assuming that TC_{it} is correctly measuring the total cost incurred by the firms, and constant return to scale, that is $MC_{it} = TC_{it}/Y_{it}$, implies that the Lerner index is equal to the price-cost margin $Lerner_{it} = \frac{P_{it} - \frac{TC_{it}}{Y_{it}}}{P_{it}} = \frac{P_{it} - MC_{it}}{P_{it}}$. This "price-cost margin" measure is then transformed into a measure of markup by taking $\mu_{it}^{Lerner} = (1 - Lerner_{it})^{-1}$, i.e. the inverse ratio of total cost to revenue $\mu_{it}^{Lerner} = \frac{P_{it}Y_{it}}{TC_{it}}$.

The results of this exercise are collected in the column "Lerner" of Table A3. All the results are qualitatively the same as in our baseline specification: the sign and significance of coefficients are identical across columns (1) and (2). This confirms that our empirical results and analysis are robust this alternative measure of markup based on the "accounting approach".

C.2   Period 2009-2019

In this robustness exercise, we restrict our sample to the period 2009-2019. Recall that the sub-sample used to estimate the production function elasticity starts only in 2009 as firm-level quantities and prices are not available for earlier years. In our baseline, in order to maximize the sample of markups available for our exercises, we assumed that the estimated production

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[A60]The expenditure on capital is computed assuming capital return net of depreciation of 4%. Here we abstract from risk or sector heterogeneity in depreciation rate.

Table A3: Robustness Table

	(1) Baseline	(2) Lerner	(3) 2009-2019	(4) Winsorized at 1%	(5) Revenue	(6) Single-Product	(7) Estimation Sample	(8) Manufacturing
Firm-Level Markup and Market Share								
First-Diff.	−0.268	−0.961	−0.459	−0.350	−0.818	−0.121	−0.063	−0.223
	(0.092)	(0.073)	(0.175)	(0.151)	(0.068)	(0.056)	(0.030)	(0.065)
Sector-Level Markup and HHI								
First-Diff.	−0.354	−0.093	−0.199	−0.467	−0.0679	−0.347	−0.127	−0.096
	(0.172)	(0.047)	(0.361)	(0.190)	(0.121)	(0.133)	(0.081)	(0.172)
Within Contribution to Sector Markup Change								
Median	0.530	0.863	0.598	0.542	0.689	0.525	0.600	0.676
Standard Deviation	0.299	0.172	0.345	0.290	0.276	0.296	0.341	0.270
Firm-Level Markup and Sector Output								
ΔY_{kt}	−0.024	0.033	0.021	−0.021	0.019	0.006	0.001	0.012
	(0.009)	(0.004)	(0.012)	(0.010)	(0.004)	(0.006)	(0.005)	(0.011)
$\Delta Y_{kt} * s_{kit}$	0.280	0.127	−0.002	0.355	0.152	0.226	0.067	0.145
	(0.041)	(0.030)	(0.096)	(0.050)	(0.029)	(0.034)	(0.043)	(0.055)
Firm-Level Market Share and Sector Output								
All firms	−0.488	−0.484	−0.507	−0.520	−0.487	−0.503	−0.501	−0.507
	(0.018)	(0.018)	(0.022)	(0.019)	(0.018)	(0.022)	(0.035)	(0.018)
$s_{kit} < 0.5$	−0.489	−0.485	−0.507	−0.521	−0.488	−0.504	−0.504	−0.508
	(0.018)	(0.018)	(0.022)	(0.019)	(0.018)	(0.022)	(0.035)	(0.018)
$s_{kit} > 0.5$	0.091	0.022	0.103	0.093	0.048	0.094	−0.214	−0.006
	(0.037)	(0.059)	(0.020)	(0.038)	(0.051)	(0.033)	(0.022)	(051)
Sector-Level Markup and Sector Output								
ΔY_{kt}	0.248	0.0788	0.176	0.254	0.119	0.227	0.135	0.226
	(0.0645)	(0.014)	(0.094)	(0.060)	(0.048)	(0.062)	(0.067)	(0.089)
Sector-Level HHI and Sector Output								
ΔY_{kt}	0.332	0.317	0.385	0.332	0.317	0.338	0.008	0.298
	(0.067)	(0.067)	(0.072)	(0.067)	(0.067)	(0.071)	(0.044)	(0.093)
Sector-Level Markup and Aggregate Output								
ΔY_t	−0.371	0.019	−0.059	−0.400	−0.243	−0.149	0.105	−0.061
	(0.176)	(0.013)	(0.252)	(0.194)	(0.047)	(0.097)	(0.077)	(0.230)

NOTE: This table reproduces our baseline estimates for various robustness checks. Column (1) collects our baseline estimates discussed in the main text (in the same order as they appear: Tables 3, 5, section 4.1.2, Tables 6, 7, 8, 9 and 10. Each other column represents a robustness exercise described in this appendix. Specifically, column (2) is discussed in appendix C.1, column (3) in C.2, column (4) in C.3, column (5) in C.4, column (6) in C.5, column (7) in C.6 and column (8) in C.7.

functions are stable and extend to the earlier period 1994-2009. This exercise addresses the concern that this assumption maybe driving our results. The results of our empirical exercices are collected in the column "2009-2019" of Table A3. The results are qualitatively similar to our baseline specification. However, the coefficient of the regression of sector markup on sector level concentration is no longer significant (though the point estimate is still negative), likely due to the lower number of observations in our sector panel in this significantly shorter subsample.

C.3 Outlier treatment

In this robustness exercise, we deploy a different outlier treatment relative to our baseline. Specifically, in our baseline specification we winsorize the firm-level markup distribution at the 3% level while in column "Winsorize at 1%" we report results for a winsorization at the 1% level. We also have explored 2% and 5% levels of winsorization. The results are barely affected by these alternative outliers treatments.

C.4 Revenue Markup

In this robustness exercise, we run our empirical specification on markups calculated with output elasticities estimated without price or quantity data. This exercise has two purposes. First, revenue markups obviate the need for quantity measurement or correct for quality differences, for example, and are widely used in the literature. Second, following the conclusions of De Ridder et al. (2022), revenue-based markups should include information about the true quantity markup. In particular, note that this revenue-based markup and our baseline quantity-based markup have a correlation at 0.3 in log-levels and 0.4 in growth rate.The results for this markup specification are collected in the column (5) "Revenue" of Table A3. Again, the results are qualitatively similar to in our baseline specification in column (1). However, two coefficients switch statistical significance relative to the baseline: the sector markup-concentration relationship loses significance while the sector markup on aggregate output gains significance.

C.5 Single Product firms

The price data from the EAP database, as described in section 3.1, gives quantity and revenue information at the product level that we then aggregate at the firm-level. One source of concern is that the aggregation process from product to firms – in an environment where large firms tend to produce several products – maybe driving our results.. In this robustness exercise, we restrict the estimation sample on single-product firms to address this concern. Results are collected in the column " Single-Product" of Table A3 and are similar to the baseline esti-

mates even if the sample of firms used to estimate the production function drops to 117,737 observations.

C.6   Estimation Sample

In this robustness exercise, we focus on the estimation sample only. Specifically, each regression and aggregation from firm to sector-level is carried out on the same sample used to estimate the production function, that is, over only 220,733 observations on the period 2009-2019. Relative to our first robustness exercise above, note that we now also lose a large number of firms as the EAP estimation sample is only a representative survey for smaller firms. The results are collected in the column "Estimation Sample" of Table A3. For this exercise, while we do obtain similar sign patterns across the different regressions, the statistical significance of several coefficients is reduced. This is partly due to statistical power - we have a much smaller sample relative to baseline, both in term of number of periods and in number of observations - and partly due to measurement error in key covariates - arising from not accounting for the population of small firms - such as market shares and HHIs.

C.7   Manufacturing firms

In this robustness exercise, we focus on the subset of sectors in manufacturing that is for the 2-digit codes 13, 14, 15 ,16 ,17 18, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, and 33. Results are collected in the column "Manufacturing" of Table A3. Results are qualitatively similar, but since we lose many 5-digits sectors, some of the results lack statistical power, as for instance, the coefficient of inverse markup on concentration.

D   Additional Figures and Tables

D.1   Inspecting the mechanism

Table A4: Firm Inverse Markup and Market Share: Level

	Dependent Variable: μ_{kit}^{-1}					
Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
s_{kit}	-1.366	-1.382	-1.17	-0.469	-0.508	-0.297
	(0.112)	(0.113)	(0.132)	(0.133)	(0.137)	(0.146)
Year FE	N	Y	N	N	Y	N
Firm FE	N	N	N	Y	Y	Y
Market * Year FE	N	N	Y	N	N	Y
Observations	9,089,750	9,089,750	9,089,750	9,039,476	9,039,476	9,039,476

NOTE: μ_{kit}^{-1} is the inverse of firm i sector k gross markup in year t, and s_{kit} gives the market share of firm i in sector k. Columns (1)-(4) report empirical estimates for the FICUS-FARE (1995-2016) data. Standard errors (in parentheses) are clustered at the firm and year level. Inverse markups are winsorized at the 3% level.

Table A5: Market Share and Marginal Cost

	Dependent Variable: $\log s_{kit}$					
Coefficient	(1)	(2)	(3)	(4)	(5)	(6)
$\log mc_{it}$	-0.152	-0.153	-0.033	-0.009	-0.009	-0.008
	(0.013)	(0.013)	(0.011)	(0.002)	(0.002)	(0.002)
Year FE	N	Y	N	N	Y	N
Firm FE	N	N	N	Y	Y	Y
Market * Year FE	N	N	Y	N	N	Y
Observations	212,459	212,459	212,459	212,184	212,184	212,184

NOTE: $\log s_{kit}$ is the (log) firm i sector k market share, and $\log mc_{it} = \log p_{it} - \log \mu_{kit}$ is the (log) marginal cost defined as the difference between (log) price and (log) markup of of firm i in sector k at time t. Columns (1)-(4) report empirical estimates for the estimation sample FARE (2009-2019) data. We drop observations with negative markup. Standard errors (in parentheses) are clustered at the firm and year level. Variables are winsorized at the 3% level.

Table A6: Markup and Marginal Cost

Coefficient			Dependent Variable: $\log \mu_{kit}$			
	(1)	(2)	(3)	(4)	(5)	(6)
$\log mc_{it}$	-0.169	-0.169	-0.149	-0.093	-0.093	-0.096
	(0.007)	(0.007)	(0.006)	(0.007)	(0.007)	(0.008)
Year FE	N	Y	N	N	Y	N
Firm FE	N	N	N	Y	Y	Y
Market * Year FE	N	N	Y	N	N	Y
Observations	212,459	212,459	212,459	212,184	212,184	212,184

NOTE: $\log \mu_{kit}$ is the (log) firm i sector k gross markup, and $\log mc_{it} = \log p_{it} - \log \mu_{kit}$ is the (log) marginal cost defined as the difference between (log) price and (log) markup of of firm i in sector k at time t. Columns (1)-(4) report empirical estimates for the estimation sample FARE (2009-2019) data. We drop observations with negative markup. Standard errors (in parentheses) are clustered at the firm and year level. Variables are winsorized at the 3% level.

Table A7: Sector Inverse Markup and Sector HHI: Level

Coefficient		Dependent Variable: μ_{kt}^{-1}			
	(1)	(2)	(3)	(4)	(5)
HHI_{kt}	-1.301	-1.306	-0.185	-0.199	-0.419
	(0.175)	(0.181)	(0.195)	(0.193)	(0.150)
Year FE	N	Y	N	Y	N
Sector FE	N	N	Y	Y	N
Sector (2 Digit) * Year FE	N	N	N	N	Y
Number of Sectors	275	275	275	275	275
Observations	6,875	6,875	6,875	6,875	6,875

NOTE: μ_{kt}^{-1} is sector k (inverse) markup in year t, HHI_{kt} is the HHI in sector k. Columns (1)-(4) report empirical estimates for the FICUS-FARE (1995-2016) data, aggregated to the sector level. Standard errors (in parentheses) are clustered at the sector and year level.

D.2 Firm-level evidence

Table A8: Firm Markup and Sector Output

	(1) Data	(2) Data	(3) Model	(4) Model
	Dependent variable: $\log(\mu_{kit})$			
$\widehat{Y}_{kt}$	0.010		-0.001	
	(0.014)			
$\widehat{Y}_{kt} * s_{kit}$	0.158		0.190	
	(0.065)			
$\widehat{Y}_{kt}^{HP}$		-0.022		-0.002
		(0.021)		
$\widehat{Y}_{kt}^{HP} * s_{kit}$		0.413		0.425
		(0.114)		
Firm FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Obs.	8,361,273	9,039,476	-	-

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t. $\widehat{Y}_{kt}$ (resp. $\widehat{Y}_{kt}^{HP}$) is (log) value-added of sector k at time t detrended following Hamilton (2018) (resp. using a HP-filter). Columns (1) and (2) report empirical estimates for the FICUS-FARE (1995-2019) data. Standard errors are two-way clustered at the sector×year level. Columns (3) and (4) report estimates based on model-simulated data. Log markup are winsorized at the 3% level. Note that the number of observations for the deviation from a Hamilton (2018) trend is lower, as we lose a few periods due to the filtering.

Table A9: Firm Market Share and Sector Output

	(1) Data (all data)	(2) Data ($\bar{s}_{ki} < 0.50$)	(3) Data ($\bar{s}_{ki} > 0.50$)	(4) Model (all data)	(5) Model ($\bar{s}_{ki} < 0.50$)	(6) Model ($\bar{s}_{ki} > 0.50$)
Dependent variable:			$\log s_{kit}$			
$\widehat{Y}_{kt}$	-0.486	-0.488	0.143	-1.377	-1.381	0.336
	(0.027)	(0.027)	(0.052)			
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Number of Obs.	8,361,273	8,360,864	440	-	-	-
Dependent variable:			$\log s_{kit}$			
$\widehat{Y}_{kt}^{HP}$	-0.829	-0.831	0.143	-3.469	-3.477	0.343
	(0.046)	(0.046)	(0.061)			
Firm FE	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y
Number of Obs.	9,039,476	9,039,036	440	-	-	-

NOTE: $\log s_{kit}$ gives the (log) market share of firm i in sector k, year t. $\widehat{Y}_{kt}$ (resp. $\widehat{Y}_{kt}^{HP}$) is (log) value-added of sector k at time t detrended following Hamilton (2018) (resp. using a HP-filter). $\bar{s}_{ki}$ is the average market share of firm i in market k. Column (1-3) reports empirical estimates for the FICUS-FARE (1995-2019) data. Sectors are defined at the 5-digit NAF sector classification level. Column (4-6) reports estimates based on model-simulated data. Standard errors in the data are two-way clustered at the sector×year level. First-difference in log market share are winsorized at the 3% level. Note that the number of observations for the deviation from a Hamilton (2018) trend is lower, as we lose a few periods due to the filtering.

D.3 Sector-Level

Table A10: Sector Markup and Sector Output

Dependent variable:	(1) $\log \mu_{kt}$	(2) $\widehat{\log}\mu_{kt}^{HP}$	(3) $\log \mu_{kt}$	(4) $\widehat{\log}\mu_{kt}^{HP}$
	Data		Model	
Y_{kt}	0.134		0.145	
	(0.040)		(0.023)	
$\widehat{Y}_{kt}^{HP}$		0.250		0.110
		(0.065)		(0.040)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,875	6,875	6,875

NOTE: Regression of sector-level (log) level and HP-trend deviation of markup ($\log \mu_{kt}$, $\widehat{\log}\mu_{kt}^{HP}$ resp.) on sector value-added (Y_{kt}, $\widehat{Y}_{kt}^{HP}$ resp.). Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Columns (3-4) reports estimates based on model-simulated data. The point estimate for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A11: Sector Concentration and Sector Output

Dependent variable:	(1) Data $\log HHI_{kt}$	(2) Data $\widehat{\log}HHI_{kt}^{HP}$	(3) Model $\log HHI_{kt}$	(4) Model $\widehat{\log}HHI_{kt}^{HP}$
Y_{kt}	0.094		1.258	
	(0.046)		(0.292)	
$\widehat{Y}_{kt}^{HP}$		0.330		0.554
		(0.064)		(0.241)
Sector FE	Y	Y	Y	Y
Year FE	Y	Y	Y	Y
Number of Sectors	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log markup on sector (log) value-added in level and HP-trend deviation ($\log HHI_{kt}$, Y_{kt} and $\widehat{\log}HHI_{kt}^{HP}$, $\widehat{Y}_{kt}^{HP}$ resp.). Column (1-2) reports empirical estimates for the FICUS-FARE (1995-2019) data, and standard errors (in parentheses) are clustered at the sector level. Columns (3-4) reports estimates based on model-simulated data. The point estimate for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A12: Sector Markup and Aggregate Output

Dependent variable:	(1) Data $\widehat{\log}\mu_{kt}^{HP}$	(2) Model without Aggr. Shocks $\widehat{\log}\mu_{kt}^{HP}$	(3) Model with Aggr. Shocks $\widehat{\log}\mu_{kt}^{HP}$
$\widehat{Y}_t^{HP}$	-0.371	0.165	0.015
	(0.227)	(0.108)	(0.044)
Share negative coefficients	-	0.02	0.20
Sector FE	Y	Y	Y
Number of Sectors	275	275	275
Number of Obs.	6,875	6,875	6,875

NOTE: Regression of sector k's markup in year t in HP trend deviation $\widehat{\log}\mu_{kt}^{HP}$ on (log) aggregate real value-added in year t in HP trend deviation $\widehat{Y}_t^{HP}$. Columns (1) report empirical estimates for the FICUS-FARE (1995-2019) data. Standard errors (in parentheses) are clustered at the year level. Columns (2) report estimates based on model-simulated data. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same samples. Column (3) reports estimates based on model-simulated data with aggregate TFP shocks. Point estimates and standard deviation for this column is computed as for columns (2). The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output measured in deviation from HP trend in France. Regression are weighted by average sector value-added.
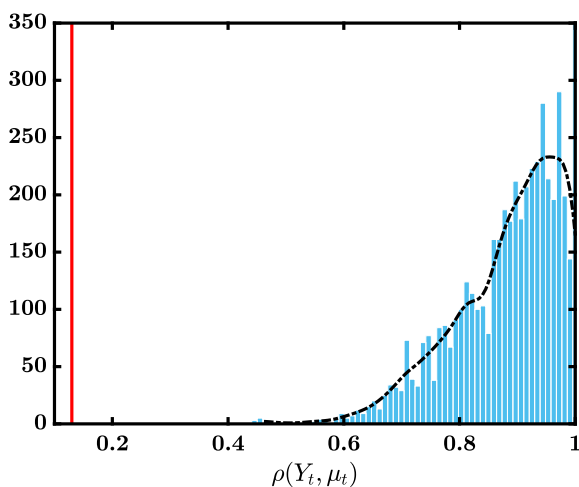
D.4 Aggregate-level

Table A13: Aggregate Markup and Aggregate Output

	(1) Data			(2) Model			(3) Model with Aggr. Shock		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\widehat{Y}_t^{HP}$	1.81	1	1	0.54	1	1	1.81	1	1
$\widehat{\mu}_t^{HP}$	1.39	0.76	0.08	0.19	0.36	0.90	0.24	0.13	0.38

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for aggregate output $\widehat{Y}_t^{HP}$ and aggregate markup $\widehat{\mu}_t^{HP}$ in deviations from their HP trend. Column (1) reports empirical estimates for the FICUS-FARE (1995-2019) data. Column (2) reports the median over 5,000 independent simulated samples, each of 25 years. Column (3) reports the average over 5,000 simulated samples of 25 years from a model with aggregate TFP shocks. The volatility of the serially uncorrelated aggregate TFP shocks is calibrated to match the aggregate volatility of aggregate output measured in deviation from HP trend in France.

Panel A: Correlation   Panel B: Ratio of standard deviations
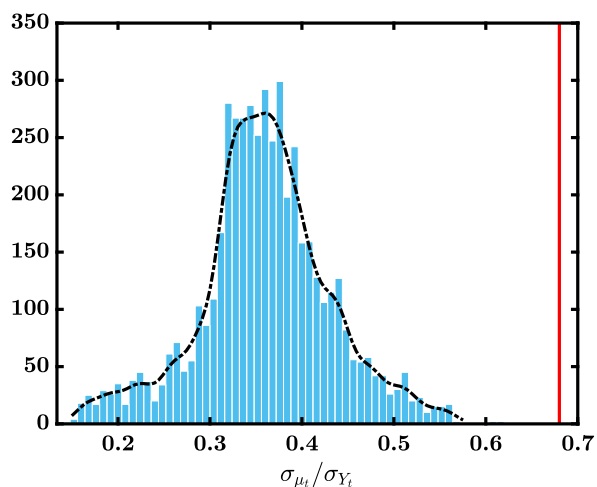
Figure A1: Histogram of Correlation and Relative Standard Deviations of Aggregate Markups and Output in Model-Simulated Data

NOTE: Kernel density of $\rho(\Delta\mu_t, \Delta Y_t)$, the correlation coefficient between aggregate markups and aggregate output, and $\sigma(\Delta\mu_t)/\sigma(\Delta Y_t)$, the ratio of standard deviation of aggregate markups and aggregate output, on model-simulated data based on 5,000 repetitions of 25 period samples. Vertical redlines show the empirical estimates.

E   Alternative Calibration Results

In this section, we reproduce the quantitative results for various alternative calibration of the preference parameters ε. For each calibration, we choose the remaining parameters to match the same targets of Table 1 as in our baseline calibration with $\varepsilon = 5$.

Table A14: Firm Markup and Sector Output

	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$	(2)	(3) $(\sigma = 1.92$ and $\varepsilon = 6)$	(4)	(5) $(\sigma = 1.80$ and $\varepsilon = 5)$	(6)	(7) $(\sigma = 1.66$ and $\varepsilon = 4)$	(8)
Dependent variable:	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$	$\log(\mu_{kit})$	$\Delta\log(\mu_{kit})$
Y_{kt}	-0.001		-0.001		-0.001		-0.001	
$Y_{kt} * s_{kit}$	0.272		0.236		0.265		0.264	
ΔY_{kt}		-0.001		-0.001		-0.001		-0.001
$\Delta Y_{kt} * s_{kit}$		0.281		0.227		0.247		0.248
Firm FE	Y	N	Y	N	Y	N	Y	N
Year FE	Y	N	Y	N	Y	N	Y	N

NOTE: μ_{kit} is firm i sector k gross markup in year t, s_{kit} gives the market share of firm i in sector k, year t and Y_{kt} sector k's (log) value-added in year t. $\Delta\log(\mu_{kit})$ is the first-difference of (log) gross markup in year t for firm i sector k, s_{kit} gives the market share of firm i in sector k, year t and ΔY_{kt} is the first-difference of sector k (log) value-added in year t. All columns report estimates based on model-simulated data for various choices of elasticities σ and ε.

Table A15: Firm Market Share and Sector Output

	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$			(2) $(\sigma = 1.92$ and $\varepsilon = 6)$			(3) $(\sigma = 1.8$ and $\varepsilon = 5)$			(4) $(\sigma = 1.66$ and $\varepsilon = 4)$		
	all	small	large	all	small	large	all	small	large	all	small	large
Dep. var.						$\log s_{kit}$						
Y_{kt}	-3.404	-3.419	0.583	-2.890	-2.900	0.273	-2.613	-2.621	0.535	-1.977	-1.979	0.146
Firm FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y
Dep. var.						$\Delta\log s_{kit}$						
ΔY_{kt}	-3.404	-3.412	0.355	-2.925	-2.932	0.253	-2.585	-2.591	0.274	-1.952	-1.956	0.283
Firm FE	N	N	N	N	N	N	N	N	N	N	N	N
Year FE	N	N	N	N	N	N	N	N	N	N	N	N

NOTE: s_{kit} gives the market share of firm i in sector k, year t, and Y_{kt} is the deviation of sector k (log) value-added in year t from its mean. $\Delta\log s_{kit}$ gives the first-difference of (log) market share of firm i in sector k, year t, and ΔY_{kt} is the first-difference of sector k (log) value-added in year t. $\bar{s}_{ki}$ is the average market share of firm i in market k. All columns report estimates based on model-simulated data for various choices of elasticities σ and ε.

Table A16: Sector Markup and Sector Output

	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$	(2)	(3) $(\sigma = 1.92$ and $\varepsilon = 6)$	(4)	(5) $(\sigma = 1.8$ and $\varepsilon = 5)$	(6)	(7) $(\sigma = 1.66$ and $\varepsilon = 4)$	(8)
Dependent variable:	$\Delta \log \mu_{kt}$	$\widehat{\log \mu}_{kt}$	$\Delta \log \mu_{kt}$	$\widehat{\log \mu}_{kt}$	$\Delta \log \mu_{kt}$	$\widehat{\log \mu}_{kt}$	$\Delta \log \mu_{kt}$	$\widehat{\log \mu}_{kt}$
ΔY_{kt}	0.091 (0.035)		0.105 (0.041)		0.110 (0.040)		0.103 (0.046)	
$\widehat{Y}_{kt}$		0.096 (0.032)		0.110 (0.037)		0.117 (0.035)		0.120 (0.039)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1, 3, 5 and 7), and Hamilton (2018) trend deviation of markup (columns 2, 4, 6 and 8), ($\Delta \log \mu_{kt}$, $\widehat{\log \mu}_{kt}$ resp.) on sector value-added (ΔY_{kt}, $\widehat{Y}_{kt}$ resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. The point estimates for these column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A17: Sector Concentration and Sector Output

Dependent variable:	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$ $\Delta \log HHI_{kt}$	(2) $\widehat{\log}HHI_{kt}$	(3) $(\sigma = 1.92$ and $\varepsilon = 6)$ $\Delta \log HHI_{kt}$	(4) $\widehat{\log}HHI_{kt}$	(5) $(\sigma = 1.8$ and $\varepsilon = 5)$ $\Delta \log HHI_{kt}$	(6) $\widehat{\log}HHI_{kt}$	(7) $(\sigma = 1.66$ and $\varepsilon = 4)$ $\Delta \log HHI_{kt}$	(8) $\widehat{\log}HHI_{kt}$
ΔY_{kt}	0.431 (0.193)		0.455 (0.213)		0.533 (0.235)		0.548 (0.346)	
$\widehat{Y}_{kt}$		0.530 (0.214)		0.565 (0.259)		0.726 (0.288)		0.737 (0.356)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector-level (log) change (columns 1, 3, 5 and 7), and Hamilton (2018) trend deviation of HHI (columns 2, 4, 6 and 8), ($\Delta \log HHI_{kt}$, $\widehat{\log}HHI_{kt}$ resp.) on sector value-added (ΔY_{kt}, $\widehat{Y}_{kt}$ resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. The point estimates for these column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A18: Sector Markup and Aggregate Output

Dependent variable:	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$ $\Delta \log \mu_{kt}$	(2) $\widehat{\log}\mu_{kt}$	(3) $(\sigma = 1.92$ and $\varepsilon = 6)$ $\Delta \log \mu_{kt}$	(4) $\widehat{\log}\mu_{kt}$	(5) $(\sigma = 1.8$ and $\varepsilon = 5)$ $\Delta \log \mu_{kt}$	(6) $\widehat{\log}\mu_{kt}$	(7) $(\sigma = 1.66$ and $\varepsilon = 4)$ $\Delta \log \mu_{kt}$	(8) $\widehat{\log}\mu_{kt}$
ΔY_{t}	0.140 (0.104)		0.138 (0.102)		0.165 (0.101)		0.169 (0.095)	
$\widehat{Y}_{t}$		0.144 (0.107)		0.146 (0.106)		0.169 (0.119)		0.171 (0.099)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector k's markup in year t in first-differences ($\Delta \log \mu_{kt}$, in columns 1, 3, 5 and 7) and Hamilton (2018) trend deviation ($\widehat{\log}\mu_{kt}$, in columns 2, 4, 6 and 8+) on (log) aggregate real value-added in year t in either first-differences or Hamilton (2018) trend deviation (ΔY_t and $\widehat{Y}_t$, resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) as the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A19: Sector Markup and Aggregate Output with Aggregate Shocks

Dependent variable:	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$ $\Delta \log \mu_{kt}$	(2) $\widehat{\log}\mu_{kt}$	(3) $(\sigma = 1.92$ and $\varepsilon = 6)$ $\Delta \log \mu_{kt}$	(4) $\widehat{\log}\mu_{kt}$	(5) $(\sigma = 1.8$ and $\varepsilon = 5)$ $\Delta \log \mu_{kt}$	(6) $\widehat{\log}\mu_{kt}$	(7) $(\sigma = 1.66$ and $\varepsilon = 4)$ $\Delta \log \mu_{kt}$	(8) $\widehat{\log}\mu_{kt}$
ΔY_t	0.005 (0.012)		0.006 (0.022)		0.008 (0.042)		0.012 (0.035)	
$\widehat{Y}_t$		0.010 (0.016)		0.010 (0.027)		0.017 (0.044)		0.022 (0.036)
Sector FE	Y	Y	Y	Y	Y	Y	Y	Y
Year FE	Y	Y	Y	Y	Y	Y	Y	Y
Number of Sectors	275	275	275	275	275	275	275	275
Number of Obs.	6,875	6,325	6,875	6,325	6,875	6,325	6,875	6,325

NOTE: Regression of sector k's markup in year t in first-differences ($\Delta \log \mu_{kt}$, in columns 1, 3, 5 and 7) and Hamilton (2018) trend deviation ($\widehat{\log}\mu_{kt}$, in columns 2, 4, 6 and 8+) on (log) aggregate real value-added in year t in either first-differences or Hamilton (2018) trend deviation (ΔY_t and $\widehat{Y}_t$, resp.). All columns report estimates based on model-simulated data for various choices of elasticities σ and ε. Point estimates for this column give the median coefficient obtained from running the reduced-form regression over 5,000 simulated samples, each of the same length (25 years) with aggregate productivity shocks chosen to match the aggregate volatility of output in the French data. The standard errors (in parentheses) are computed over the same simulated samples.

Table A20: Aggregate Markup and Aggregate Output

	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$ σ_x	σ_x/σ_Y	$\rho(x,Y)$	(2) $(\sigma = 1.92$ and $\varepsilon = 6)$ σ_x	σ_x/σ_Y	$\rho(x,Y)$	(3) $(\sigma = 1.8$ and $\varepsilon = 5)$ σ_x	σ_x/σ_Y	$\rho(x,Y)$	(4) $(\sigma = 1.66$ and $\varepsilon = 4)$ σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\widehat{Y}_t$	0.71	1	1	0.77	1	1	0.83	1	1	1.04	1	1
$\widehat{\mu}_t$	0.25	0.35	0.93	0.30	0.39	0.93	0.30	0.36	0.91	0.36	0.35	0.90
$\Delta \widehat{Y}_t$	0.63	1	1	0.64	1	1	0.69	1	1	0.86	1	1
$\Delta \mu_t$	0.21	0.33	0.95	0.24	0.38	0.94	0.25	0.36	0.91	0.30	0.35	0.91

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for deviations from trend computed as in Hamilton (2018) of (log) aggregate output $\widehat{Y}_t$ and (log) aggregate markup $\widehat{\mu}_t$, and, for log first-difference of aggregate output ΔY_t and aggregate markup $\Delta \mu_t$. Column (1-4) reports the median over 5,000 simulated samples of 25 years for each alternative calibration.

Table A21: Aggregate Markup and Aggregate Output with Aggregate Shocks

	(1) $(\sigma = 2.01$ and $\varepsilon = 7)$			(2) $(\sigma = 1.92$ and $\varepsilon = 6)$			(3) $(\sigma = 1.8$ and $\varepsilon = 5)$			(4) $(\sigma = 1.66$ and $\varepsilon = 4)$		
	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$	σ_x	σ_x/σ_Y	$\rho(x,Y)$
$\widehat{Y}_t$	3.16	1	1	3.16	1	1	3.16	1	1	3.16	1	1
$\widehat{\mu}_t$	0.25	0.08	0.25	0.25	0.08	0.22	0.30	0.09	0.29	0.36	0.11	0.35
$\Delta\widehat{Y}_t$	3.28	1	1	3.28	1	1	3.28	1	1	3.28	1	1
$\Delta\mu_t$	0.21	0.06	0.21	0.25	0.08	0.22	0.26	0.08	0.27	0.30	0.09	0.29

NOTE: The table reports standard deviations, σ_x, relative standard deviations, σ_x/σ_Y, and time-series correlations, $\rho(x,Y)$, for deviations from trend computed as in Hamilton (2018) of (log) aggregate output $\widehat{Y}_t$ and (log) aggregate markup $\widehat{\mu}_t$, and, for log first-difference of aggregate output ΔY_t and aggregate markup $\Delta\mu_t$. Column (1-4) reports the median over 5,000 simulated samples of 25 years for each alternative calibration with aggregate productivity shocks chosen to match the aggregate volatility of output in the French data.