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CONVERGENCE IN THE TRENDS AND CYCLES OF EURO-ZONE INCOME

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SUMMARY

Multivariate unobserved components (structural) time series models are fitted to annual post-war observations on real income per capita in countries in the Euro-zone. The aim is to establish stylized facts about convergence as it relates both to long-run and short-run movements. A new model, in which convergence components are combined with a common trend and similar cycles, is proposed. The convergence components are formulated as a second-order error correction mechanism; this ensures that the extracted components change smoothly, thereby enabling them to be separated from transitory cycles. Copyright © 2005 John Wiley & Sons, Ltd.

1. INTRODUCTION

Minimizing income inequality across its members has long been a declared objective of the European Union. In particular, promoting convergence of income in levels (or at least the conditions for that to be obtained) has been the justification for the establishment first, of Structural Funds (the largest of which is the European Regional Development Fund established in 1975) and, more recently (1993), of the Cohesion Fund. The latter is specifically directed at poorer countries (Greece, Ireland, Portugal and Spain) and was set up with the purpose of making compatible the EMU budgetary discipline requirements and the continued infrastructure investment requirements in these countries, which the EU deems necessary for convergence to take place. As Boldrin and Canova (2001) point out, what this implies about the view underlying EU policies is that deepening economic and monetary integration, by itself, leads to divergence of income levels across Europe; see also Martin (2001). However, this view stands at odds with neoclassical growth theories. As economic and monetary integration deepens and free movement of goods, people and capital becomes a reality across Europe, the preconditions for such theories are more likely to be met. As such, the theoretical prediction would be one of convergence not divergence.

Which view is correct is not only of theoretical interest but also has important policy implications. If the EU view is correct, the creation of EMU means that incentives for structural adjustment are needed for catching up to take place. Given the loss of national monetary policies and the strict budgetary requirements limiting national public spending, European-wide redistribution would be the only option. Indeed, a substantial part of the EU's resources is already directed at sustaining cohesion: Boldrin and Canova (2001) find that, over the 1986–1999 period, it is close to 8% of the Community's GDP.

Given such vast policy implications, the need to verify the validity of the underlying premise is obvious. Stylized facts on growth and convergence are needed to answer questions such as: are

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Euro-zone economies converging to a single steady state distribution or are they clustering around different states? Were they diverging before the establishment of Structural and Cohesion funds? Is there any visible effect of the latter on the growth dynamics of poorer countries?

However, the fact remains that the existing literature on European convergence—mirroring the situation throughout the entire growth empirics literature–has reached no agreement on what the European record really is. This stems mainly from the use of different methods; see Durlauf and Quah (1999) for a review and criticism of the empirical literature. Thus while the early cross-sectional approach of Barro and Sala-i-Martin (1992) and Sala-i-Martin (1996) concluded in favour of absolute but slow convergence in Europe, some panel models have suggested very fast convergence towards different steady states. Finally, following the time series/cointegration approach proposed in Bernard and Durlauf (1995, 1996), Tsionas (2000) applies a battery of unit root/stationarity tests to conclude that both divergence and convergence are possible: 'the results are mixed and depend critically on the type of test employed'.

Our view is that a clear presentation of the stylized facts on growth and convergence is of most value for the development of meaningful theories and for policy decisions. This response is in line with the growing dissatisfaction on the current state of growth empirics and Durlauf's (2001, p. 68) call for econometrics to 'clarify how empirical workers should elucidate data patterns and draw inferences concerning growth'. The aim of the present paper is to establish stylized facts about convergence in Euro-zone countries, both with respect to long-run income levels and to cycles. Distinguishing trends from cyclical movements is essential to an effective study of convergence. The analysis is based on a new multivariate unobserved components model in which convergence components are combined with a common trend and similar cycles. These convergence components are formulated as a second-order error correction mechanism which ensures that the extracted components change smoothly, thereby giving a clearer decomposition into long-run movements and cycles. Furthermore, the second-order mechanism is able to capture temporary divergence, something that is a feature of the Euro-zone data. Because the cross-section is relatively small, we are able to properly account for the cross-correlations across countries.

The plan of the paper is as follows. In Section 2 we show how a multivariate structural time series model handles balanced growth and then extend it to incorporate convergence components. Following the preliminary analysis of trends in per capita income in the Euro-zone countries in Section 3, the convergence model is fitted to two groups in Section 4. The growth paths of the two groups are then compared. Section 5 examines the evidence for convergence in cycles. Section 6 assesses the extent to which unit root and cointegration tests can provide meaningful evidence on convergence, while Section 7 concludes.

2. STRUCTURAL TIME SERIES MODELS, BALANCED GROWTH AND CONVERGENCE

Structural time series models are formulated in terms of unobserved components that have a direct interpretation. The statistical treatment of such models is based on the state space form (SSF). Once a model has been put in SSF, the Kalman filter yields estimators of the components based on current and past observations. Signal extraction, or smoothing, refers to estimation of components based on all the information in the sample. Predictions are made by extending the Kalman filter forward. The unknown parameters are estimated by constructing a likelihood function from the one-step-ahead prediction errors produced by the Kalman filter and maximizing it by an iterative procedure. The model described in the first subsection is a standard one that can be handled by the

STAMP package of Koopman *et al.* (2000). The new convergence model set out in subsection 2.3 was estimated by writing a program in the OX 3.0 language of Doornik (1999) and making use of the SSfPack collection of state space algorithms described in Koopman *et al.* (1999).

2.1. Multivariate Structural Time Series Models

Let $\mathbf{y}_t = (y_{1t}, \ldots, y_{Nt})'$ denote the vector of observations for N time series. A multivariate structural time series model may be set up in terms of trend, cycle and irregular unobserved components, denoted by $N \times 1$ vectors $\boldsymbol{\mu}_t$, $\boldsymbol{\psi}_t$ and $\boldsymbol{\varepsilon}_t$, as follows:

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \boldsymbol{\psi}_t + \boldsymbol{\varepsilon}_t, \qquad \boldsymbol{\varepsilon}_t \sim \mathrm{NID}(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon), \qquad t = 1, \ldots, T \tag{1}$$

where $\boldsymbol{\Sigma}_\varepsilon$ is an $N \times N$ positive semidefinite matrix. The (local linear) trend is

$$\begin{aligned} \boldsymbol{\mu}_t &= \boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1} + \boldsymbol{\eta}_t, \qquad \boldsymbol{\eta}_t \sim \mathrm{NID}(\mathbf{0}, \boldsymbol{\Sigma}_\eta) \\ \boldsymbol{\beta}_t &= \boldsymbol{\beta}_{t-1} + \boldsymbol{\zeta}_t, \qquad \boldsymbol{\zeta}_t \sim \mathrm{NID}(\mathbf{0}, \boldsymbol{\Sigma}_\zeta) \end{aligned} \tag{2}$$

When $\boldsymbol{\Sigma}_\zeta = \mathbf{0}$, $\boldsymbol{\mu}_t$ reduces to a multivariate random walk plus drift. With $\boldsymbol{\Sigma}_\eta = \mathbf{0}$, but $\boldsymbol{\Sigma}_\zeta$ positive definite, we get an integrated random walk. The trend extracted from the integrated random walk tends to be smoother and the model allows a clearer separation into trend and cycle.

The *similar cycle* model, introduced by Harvey and Koopman (1997), is

$$\begin{bmatrix} \boldsymbol{\psi}_t \\ \boldsymbol{\psi}_t^* \end{bmatrix} = \left[\rho \begin{pmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{pmatrix} \otimes \mathbf{I}_N \right] \begin{bmatrix} \boldsymbol{\psi}_{t-1} \\ \boldsymbol{\psi}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \boldsymbol{\kappa}_t \\ \boldsymbol{\kappa}_t^* \end{bmatrix}, \qquad t = 1, \ldots, T \tag{3}$$

where $\boldsymbol{\psi}_t$ and $\boldsymbol{\psi}_t^*$ are $N \times 1$ vectors and $\boldsymbol{\kappa}_t$ and $\boldsymbol{\kappa}_t^*$ are $N \times 1$ vectors of disturbances such that

$$E(\boldsymbol{\kappa}_t\boldsymbol{\kappa}_t') = E(\boldsymbol{\kappa}_t^*\boldsymbol{\kappa}_t^{*'}) = \boldsymbol{\Sigma}_\kappa, \qquad E(\boldsymbol{\kappa}_t\boldsymbol{\kappa}_t^{*'}) = \mathbf{0} \tag{4}$$

where $\boldsymbol{\Sigma}_\kappa$ is an $N \times N$ covariance matrix. The model allows the disturbances to be correlated across the series. Because the damping factor and the frequency, ρ and λ_c, are the same in all series, the cycles in the different series have the same spectral density.

2.2. Stability and Balanced Growth

The *balanced growth* unobserved components model is a special case of (1):

$$\mathbf{y}_t = \mathbf{i}\mu_t + \boldsymbol{\alpha} + \boldsymbol{\psi}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, \ldots, T \tag{5}$$

where μ_t is a univariate local linear trend, $\mathbf{i}$ is a vector of ones and $\boldsymbol{\alpha}$ is an $N \times 1$ vector of constants. If μ_t is initialized with a diffuse prior, then $\boldsymbol{\alpha}$ must be subject to a constraint so it contains only $N - 1$ free parameters. Note that although the levels may be different across series, the slopes are the same, irrespective of whether they are fixed or stochastic.

A balanced growth model implies that the series has a stable relationship over time, in that the difference between any pair is stationary.

2.3. Convergence Models

A multivariate convergence model may be set up as

$$\mathbf{y}_t = \boldsymbol{\alpha} + \beta \mathbf{i} t + \boldsymbol{\mu}_t + \boldsymbol{\psi}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, \ldots, T \tag{6}$$

with $\boldsymbol{\alpha}$, $\boldsymbol{\psi}_t$ and $\boldsymbol{\varepsilon}_t$ defined as (1) and

$$\boldsymbol{\mu}_t = \boldsymbol{\Phi}\boldsymbol{\mu}_{t-1} + \boldsymbol{\eta}_t, \qquad \mathrm{Var}(\boldsymbol{\eta}_t) = \boldsymbol{\Sigma}_\eta \tag{7}$$

Each row of $\boldsymbol{\Phi}$ sums to unity, that is $\boldsymbol{\Phi}\mathbf{i} = \mathbf{i}$. Thus setting λ to one in $(\boldsymbol{\Phi} - \lambda\mathbf{I})\mathbf{i} = \mathbf{0}$ shows that $\boldsymbol{\Phi}$ has an eigenvalue of one with a corresponding eigenvector consisting of ones. The other roots of $\boldsymbol{\Phi}$ are obtained by solving $|\boldsymbol{\Phi} - \lambda\mathbf{I}| = 0$; they should have modulus less than one for convergence.

If we write

$$\overline{\boldsymbol{\phi}}'\boldsymbol{\mu}_t = \overline{\boldsymbol{\phi}}'\boldsymbol{\Phi}\boldsymbol{\mu}_{t-1} + \overline{\boldsymbol{\phi}}'\boldsymbol{\eta}_t$$

it is clear that the $N \times 1$ vector of weights, $\overline{\boldsymbol{\phi}}$, which gives a random walk must be such that $\overline{\boldsymbol{\phi}}'\boldsymbol{\Phi} = \overline{\boldsymbol{\phi}}'$. Since the roots of $\boldsymbol{\Phi}'$ are the same as those of $\boldsymbol{\Phi}$, it follows from writing $\boldsymbol{\Phi}\overline{\boldsymbol{\phi}}' = \overline{\boldsymbol{\phi}}'$ that $\overline{\boldsymbol{\phi}}$ is the eigenvector of $\boldsymbol{\Phi}'$ corresponding to its unit root. This random walk, $\overline{\mu}_{\phi,t} = \overline{\boldsymbol{\phi}}'\boldsymbol{\mu}_t$, *is a common trend in the sense that it yields the common growth path to which all the economies converge*. This is because $\lim_{j\to\infty} \boldsymbol{\Phi}^j = \mathbf{i}\overline{\boldsymbol{\phi}}'$; the proof follows along the same lines as that for a well-known result on ergodic Markov chains as given, for example, in Hamilton (1994, p. 681). The common trend for the observations is a random walk with drift, β, and if the restriction on $\boldsymbol{\alpha}$ is imposed by requiring that $\boldsymbol{\alpha}'\overline{\boldsymbol{\phi}} = 0$, each element of $\boldsymbol{\alpha}$ is a deviation from the common trend.

The *homogeneous* model has $\boldsymbol{\Phi} = \phi\mathbf{I} + (1-\phi)\mathbf{i}\overline{\boldsymbol{\phi}}'$, where $\mathbf{i}$ is an $N \times 1$ vector of ones, ϕ is a scalar convergence parameter and $\overline{\boldsymbol{\phi}}$ is an $N \times 1$ vector of parameters with the property that $\overline{\boldsymbol{\phi}}'\mathbf{i} = 1$. (It is straightforward to confirm that $\overline{\boldsymbol{\phi}}$ is the eigenvector of $\boldsymbol{\Phi}'$ corresponding to the unit root.) The likelihood function is maximized numerically with respect to ϕ and the elements of $\overline{\boldsymbol{\phi}}$, denoted $\overline{\phi}_i$, $i = 1, \ldots, N$; the $\boldsymbol{\mu}_t$ vector is initialized with a diffuse prior. It is assumed that $0 \le \phi \le 1$, with $\phi = 1$ indicating no convergence. The $\overline{\phi}_i'$s are constrained to lie between zero and one and to sum to one by employing a logistic transformation[1] and maximizing with respect to $N-1$ unconstrained parameters, ξ_i, $i = 1, \ldots, N-1$, defined by $\overline{\phi}_i = \overline{\phi}_i^* \exp(\xi_i)/(1+\exp(\xi_i))$, where $\overline{\phi}_i^* = 1 - \sum_{j=1}^{i-1}\overline{\phi}_j$, $j = 2, \ldots, N-1$ and $\overline{\phi}_1^* = 1$.

Each trend in a homogeneous model can be decomposed into the common trend and a convergence component. The vector of convergence components is defined by $\boldsymbol{\mu}_t^\dagger = \boldsymbol{\mu}_t - \mathbf{i}\overline{\mu}_{\phi,t}$, and it is easily seen that

$$\boldsymbol{\mu}_t^\dagger = \phi\boldsymbol{\mu}_{t-1}^\dagger + \boldsymbol{\eta}_t^\dagger, \qquad t = 1, \ldots, T \tag{8}$$

where $\boldsymbol{\eta}_t^\dagger = \boldsymbol{\eta}_t - \mathbf{i}\overline{\eta}_{\phi,t}$. The error correction form for each series

$$\Delta\mu_{it}^\dagger = (\phi - 1)\mu_{i,t-1}^\dagger + \eta_{it}^\dagger, \qquad t = 1, \ldots, T; i = 1, \ldots, N$$

shows that its relative growth rate depends on the gap between it and the common trend, $\overline{\mu}_{\phi,t}$. Substituting (8) into (6) gives

$$\mathbf{y}_t = \boldsymbol{\alpha} + \beta\mathbf{i}t + \mathbf{i}\overline{\mu}_{\phi,t} + \boldsymbol{\mu}_t^\dagger + \boldsymbol{\psi}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, \ldots, T$$

[1] We are grateful to a referee for pointing out that our original method for dealing with this problem suffered from certain drawbacks.

Once convergence has taken place, the model is of the balanced growth form (5), but with an additional stationary component $\boldsymbol{\mu}_t^\dagger$. If the state vector is defined in terms of the common trend and convergence components, only $N-1$ of the latter need be included as $\sum_{i=1}^N \overline{\phi}_i \mu_{it}^\dagger = 0$.

The smooth homogeneous convergence model is

$$\mathbf{y}_t = \boldsymbol{\alpha} + \boldsymbol{\mu}_t + \boldsymbol{\psi}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, \ldots, T \tag{9}$$

and

$$\boldsymbol{\mu}_t = \boldsymbol{\Phi}\boldsymbol{\mu}_{t-1} + \boldsymbol{\beta}_{t-1}, \qquad \boldsymbol{\beta}_t = \boldsymbol{\Phi}\boldsymbol{\beta}_{t-1} + \boldsymbol{\zeta}_t, \qquad \mathrm{Var}(\boldsymbol{\zeta}_t) = \boldsymbol{\Sigma}_\zeta \tag{10}$$

with $\boldsymbol{\Phi} = \phi\mathbf{I} + (1-\phi)\mathbf{i}\overline{\boldsymbol{\phi}}'$ as before. Using scalar notation to write the model in terms of the common trend, $\overline{\mu}_{\phi,t}$, and convergence processes, $\mu_{it}^\dagger = \mu_{it} - \overline{\mu}_{\phi,t}$, $i = 1, \ldots, N$, we obtain

$$y_{it} = \alpha_i + \overline{\mu}_{\phi,t} + \mu_{it}^\dagger + \psi_{it} + \varepsilon_{it}, \qquad t = 1, \ldots, T; i = 1, \ldots, N \tag{11}$$

where the common trend is

$$\overline{\mu}_{\phi,t} = \overline{\mu}_{\phi,t-1} + \overline{\beta}_{\phi,t-1}, \qquad \overline{\beta}_{\phi,t} = \overline{\beta}_{\phi,t-1} + \overline{\zeta}_{\phi,t}$$

and the convergence components are

$$\mu_{it}^\dagger = \phi\mu_{i,t-1}^\dagger + \beta_{it}^\dagger, \qquad \beta_{it}^\dagger = \phi\beta_{i,t-1}^\dagger + \zeta_{it}^\dagger, \qquad i = 1, \ldots, N \tag{12}$$

As in the first-order model $0 \le \phi \le 1$, with $\phi = 1$ indicating no convergence since the $\mu_{it}^\dagger$s would then have the same form as in a smooth trend model.

The convergence components in (12) can be given a second-order error correction representation

$$\Delta\mu_{it}^\dagger = (\phi - 1)\mu_{i,t-1}^\dagger + \beta_{it}^\dagger, \qquad \Delta\beta_{it}^\dagger = (\phi - 1)\beta_{i,t-1}^\dagger + \zeta_{it}^\dagger, \qquad i = 1, \ldots, N$$

Alternatively[2]

$$\Delta\mu_{it}^\dagger = -(1-\phi)^2\mu_{i,t-1}^\dagger + \phi^2\Delta\mu_{i,t-1}^\dagger + \zeta_{it}^\dagger, \qquad i = 1, \ldots, N \tag{13}$$

showing that the underlying change depends not only on the gap but also on the change in the previous time period. As a result, the extracted components typically change relatively slowly, thereby enabling them to be separated from transitory cycles. As in the first-order error correction model, the forecasts for the individual trends converge to the common trend, but in doing so they may now exhibit temporary divergence.

3. PRELIMINARY STYLIZED FACTS

In this section we use the multivariate model of subsection 2.1 to display the stylized facts concerning trends and convergence in real per capita incomes in 11 Euro-zone countries: Austria (AU), Belgium (BE), Finland (FI), France (FR), Germany (GE), Greece (GR), Ireland (IR), Italy

[2] Since the model in (13) is equivalent to an AR(2) process with both roots equal to ϕ, it is clear that the condition for stationarity is $|\phi| < 1$. The autocorrelation function is $[1 + \{(1-\phi^2)/(1+\phi^2)\}\tau]\phi^\tau$, $\tau = 0, 1, 2, \ldots$ so the decay is slower than in an AR(1) with the same value of ϕ.

(IT), The Netherlands (NE), Portugal (PO) and Spain (SP). Out of the present membership of the Euro-area, only Luxembourg is not included as no data are available.

3.1. Data and Estimation

Annual data were obtained from the GGDC Total Economy Database, 2002, at the University of Groningen and the Conference Board.[3] All series are expressed in 1990 US dollars converted at 'Geary–Khamis' purchasing power parities. We fitted models to log-transformed observations from 1950 to 1997. The German data are for West Germany. After 1997 observations are only available for Germany as a whole and that is why we concentrated our analysis on the period up to that point. In any case, having data for the period after 1997 is useful as it allows us to consider the performance of the model in the light of subsequent events.

3.2. Trends and Convergence Clubs

Fitting a multivariate smooth trend plus cycle model, (1), allows us to extract slowly changing trends from which we can start to assess any tendencies towards long-run convergence. From a study of the graph it is apparent that there are two groups within the Euro-zone: a high-income group, consisting of core countries, GE, FR, BE, NE and IT, together with AU and FI, and a low-income group made up of poorer peripheral countries, PO, SP and GR; see the discussion in Durlauf and Johnson (1995). Ireland does not fit into either group as it is relatively rich at the beginning and end of the sample and poor in the middle. The two groups are well-defined and remarkably stable throughout the second half of the twentieth century. The three poorest and the seven richest countries in 1950 still had that status at the end of the 1990s.

A second point is that the extracted trends clearly show the three distinct epochs of economic growth: 1950–1972, 1973–1979 and 1980–1997. These subdivisions in post-war European economic history correspond to the 'Golden Age', the shocks of the 1970s and the ensuing period of stabilization and restructuring; see Crafts and Toniolo (1996). The poorer group had a higher growth rate over the entire period, primarily due to the catching-up process during the Golden Age.

The above conclusions are supported by Figure 1, which shows the cross-sectional standard deviation of the smoothed trends against time for all countries and for the two groups separately. The series for all countries falls over time, particularly during the 1960s, but is much higher than the series for the two groups. The series for the rich countries shows a steady decline over the whole period. The corresponding plot for the eight United States census regions is shown for comparison; see Carvalho and Harvey (2002).

4. FITTING CONVERGENCE MODELS

The extracted trends indicate the existence of two convergence clubs, plus Ireland. We therefore proceed to fit the convergence model of subsection 2.3 to the rich and poor groups separately, rather than to the Euro-zone as a whole. Recall that the model not only allows us to separate trends from cycles, but also yields the common trend to which the economies are converging. The results

[3] For a full description of the data set refer to http://www.eco.rug.nl/ggdc.

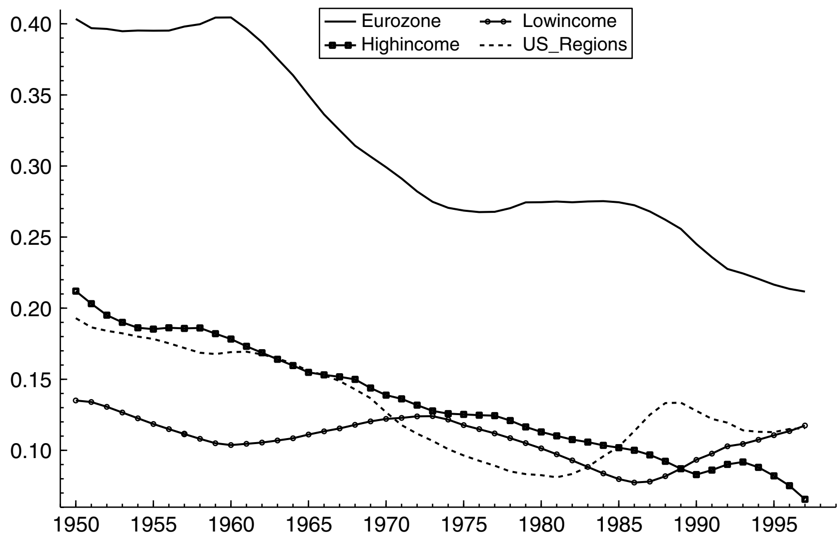

Figure 1. Standard deviations of trends in the Euro-zone and USA

reported are for the homogeneous smooth convergence model, (9), with absolute convergence, that is $\boldsymbol{\alpha} = \mathbf{0}$. Having fitted this model to the two groups we proceed to investigate the case for relative convergence of the two groups by comparing their common trends.

Covariance matrices in this section and the next are reported by showing the variances, multiplied by 10^5, on the main diagonal while the entries above contain the cross-correlations. The components in the figures are extracted by the state space smoothing algorithm.

4.1. High-Income Group

The convergence parameter, ϕ, was estimated to be 0.938 while the common trend weights, $\overline{\phi}_i$, are:

$\overline{\phi}_{AU}$	$\overline{\phi}_{BE}$	$\overline{\phi}_{FI}$	$\overline{\phi}_{FR}$	$\overline{\phi}_{GE}$	$\overline{\phi}_{IT}$	$\overline{\phi}_{NE}$
0.01	0.06	0.45	0.02	0.03	0.40	0.03

Figure 2 displays the estimate of the common trend, $\overline{\mu}_{\phi,t}$, together with the estimated trends for each country, while Figure 3 shows the smoothed estimates of the convergence components, $\mu_{it}^{\dagger}$, for the seven high-income countries.

The large weights assigned to Finland and Italy in constructing the common trend mean that the future growth path depends primarily on extrapolating the trends in these two countries. In other words these two relatively poorer countries within the high-income group act as benchmarks to which all the other high-income countries converge. This may seem surprising at first. Why doesn't Germany dominate, for example? A glance at Figure 2 gives the answer. The German growth rate has been gradually slowing down, particularly after reunification, and at the end of the sample the trend is almost flat. If we were to extrapolate there would be almost no growth.

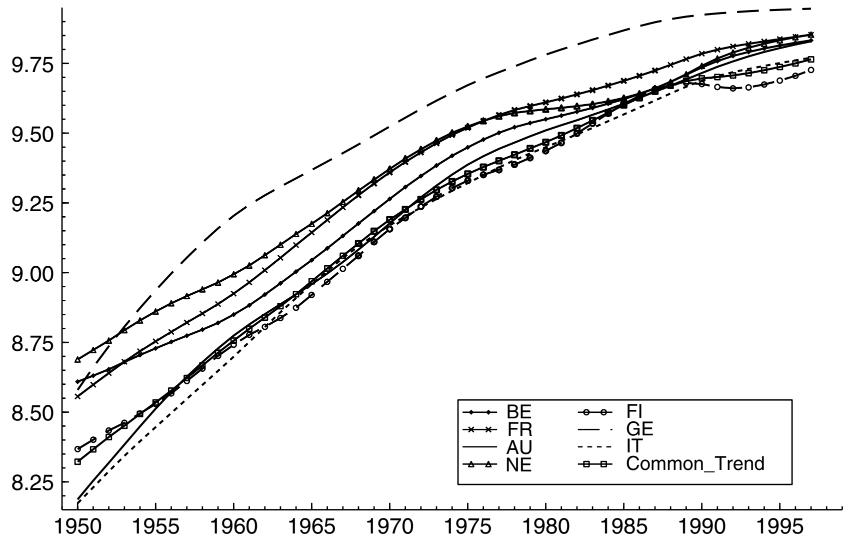

Figure 2. National trends and common trend for the high-income group

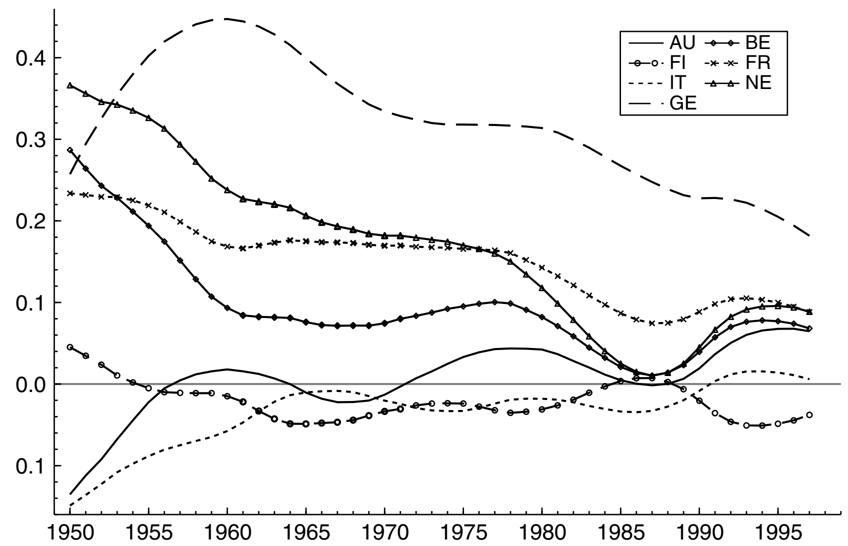

Figure 3. Convergence components for the high-income group

The convergence components are not monotonic over time and so the fact that the model allows a degree of divergence is important. During the 1950s and early 1960s at least two distinct types of dynamics may be singled out. One is dominated by the strong effects of reconstruction in Austria, Italy and Germany and the other by relatively slower adjustment in France, Belgium and The Netherlands; further details can be found in Crafts and Toniolo (1996) and the references therein. In contrast, the late 1960s and 1970s appear to bring a halt to the convergence process across the entire high-income group. This process is resumed in the 1980s before again slowing down in the 1990s. It is worth noting that these effects are not apparent from the standard deviation series of Figure 1, which shows a steady decline over the whole period.

The matrix of variances and cross-correlations obtained from the covariance matrix, $\widetilde{\boldsymbol{\Sigma}}_{\zeta}$, of the disturbances driving the converging trends is shown below. The correlations within a group formed by Belgium, France and The Netherlands are the highest.

$$\begin{array}{c} \text{AU} \\ \text{BE} \\ \text{FI} \\ \text{FR} \\ \text{GE} \\ \text{IT} \\ \text{NE} \end{array} \begin{bmatrix} 1.80 & 0.31 & 0.15 & -0.03 & 0.49 & -0.48 & 0.48 \\ & 1.22 & -0.19 & 0.83 & -0.63 & 0.09 & 0.94 \\ & & 0.82 & 0.18 & 0.57 & 0.03 & -0.26 \\ & & & 0.86 & -0.60 & 0.48 & 0.70 \\ & & & & 0.48 & -0.36 & -0.47 \\ & & & & & 1.06 & 0.15 \\ & & & & & & 2.23 \end{bmatrix}$$

The forecasts of the convergence components will tend to zero as the lead time goes to infinity. The second-order error correction mechanism allows some divergence before convergence eventually takes place. However, in Figure 3 all the convergence components are pointing in the direction of zero and a straightforward extrapolation suggests that the economies will be close to convergence after about 10 years.

4.2. Low-Income Group

We now turn to the converging dynamics within the low-income group. The convergence parameter was estimated as $\phi = 0.937$, while the common trend weights are $\overline{\phi}_{\text{GR}} = 0.48$, $\overline{\phi}_{\text{PO}} = 0.36$ and $\overline{\phi}_{\text{SP}} = 0.17$. Figure 4 shows the estimated trends.

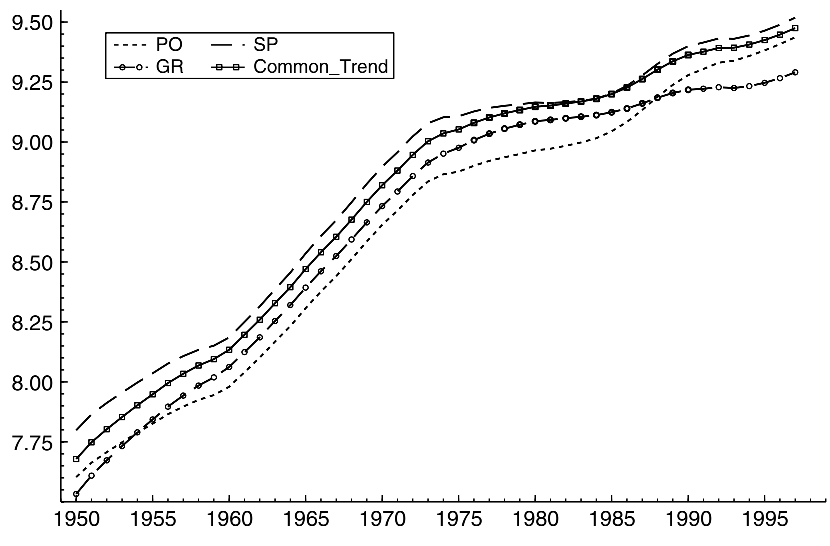

Figure 4. National trends and common trend for the low-income group

The variances and cross-correlations in the covariance matrix of the trend disturbances are:

$$\begin{array}{c} \mathrm{GR} \\ \mathrm{PO} \\ \mathrm{SP} \end{array} \begin{bmatrix} 10.0 & 0.89 & 0.95 \\ & 17.5 & 0.99 \\ & & 22.8 \end{bmatrix}$$

All the pairwise correlations are greater than the BE–FR correlation, the largest in the high-income group. The magnitude of the convergence components is smaller and more stable for these three low-income countries. However, the behaviour of the convergence components illustrates the point that a relatively stable cross-sectional standard deviation does not necessarily imply that there are no intra-distribution dynamics; see Durlauf and Quah (1999). Thus, Greece and Portugal alternate their ranking over the sample period: Greece starts off as the poorest economy, rapidly overtakes Portugal, but is subsequently overtaken by Portugal in the 1980s.

4.3. Divergence or Relative Convergence in the Euro-zone?

Having characterized the within-group convergence dynamics we now proceed with a between-groups analysis. The key question is whether the common trends estimated for each group indicate relative convergence. In order to investigate this issue, a bivariate convergence model is fitted to the two common trends. Since we are dealing with extracted trends, $\widetilde{\mu}_{i,t|T}$, the aim is to perform the decomposition

$$\tilde{\mu}_{it|T} = \tilde{\alpha}_i + \tilde{\overline{\mu}}_{\phi,t|T} + \tilde{\mu}^{\dagger}_{it|T}, \qquad i = \text{LI, HI} \tag{14}$$

where $\tilde{\overline{\mu}}_{\phi,t|T}$ and $\tilde{\mu}^{\dagger}_{it|T}$ are as implied by (11) and LI and HI denote low income and high income, respectively.

When the model is fitted, the estimate of $\overline{\phi}_{\mathrm{HI}}$ is one, indicating that the high-income trend can be treated as a benchmark following an integrated random walk. The variance of the disturbance driving this trend is 0.54×10^5, while the corresponding variance for the convergence component is 6.51×10^5. The estimate of ϕ is 0.916, which is consistent with convergence of the low-income trend towards the high-income trend. Since the high-income group is a benchmark, the difference between the two series in the long run is $\tilde{\alpha}_{\mathrm{LI}} = -0.337$. This implies that the trend forecasts of the low-income countries are converging to a growth path, $\tilde{\mu}_{\mathrm{HI},t} + \tilde{\alpha}_{\mathrm{LI}}$, that is parallel to that of the high-income countries but below it. Figure 5 shows a plot of $\tilde{\mu}_{\mathrm{HI},t} - 0.337$, together with the two group common trends. It can be seen that relative convergence had taken place by the early 1970s. The gap appears to be permanent and the long-run implication is that each of the low-income countries will have a per capita income that is almost 30% below that in the high-income group.

5. CYCLES

The other aspect of convergence concerns short-term cycles. In order to investigate this issue we focus on the five core economies of France, Germany, Belgium, The Netherlands and Italy. On fitting a multivariate model to these countries, the estimates of the damping factor, ρ, and the period were found to be 0.87 and 7.86 years, respectively. The variances and cross-correlations are shown below:

$$
\begin{array}{l}
\text{BE} \\ \text{FR} \\ \text{GE} \\ \text{NE} \\ \text{IT}
\end{array}
\begin{bmatrix}
15.1 & 0.75 & 0.79 & 0.55 & 0.28 \\
 & 12.2 & 0.78 & 0.45 & 0.34 \\
 & & 24.8 & 0.72 & 0.35 \\
 & & & 25.6 & 0.26 \\
 & & & & 31.6
\end{bmatrix}
$$

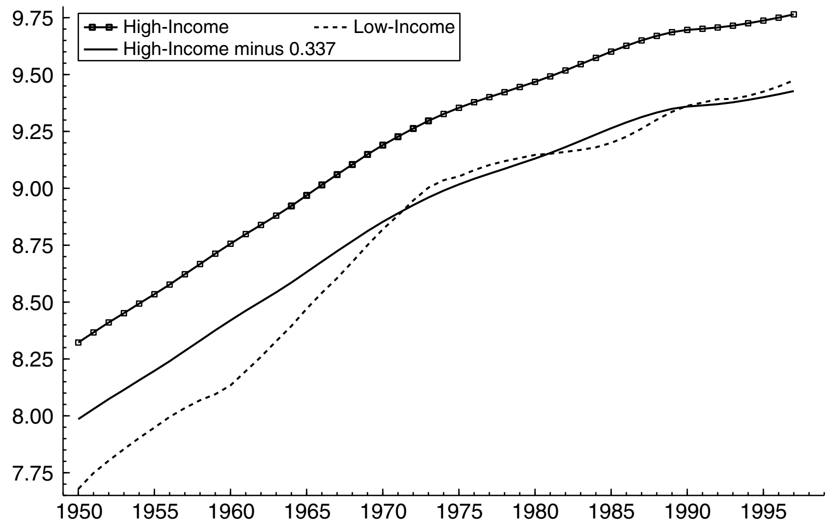

Figure 5. Common trends for high- and low-income groups

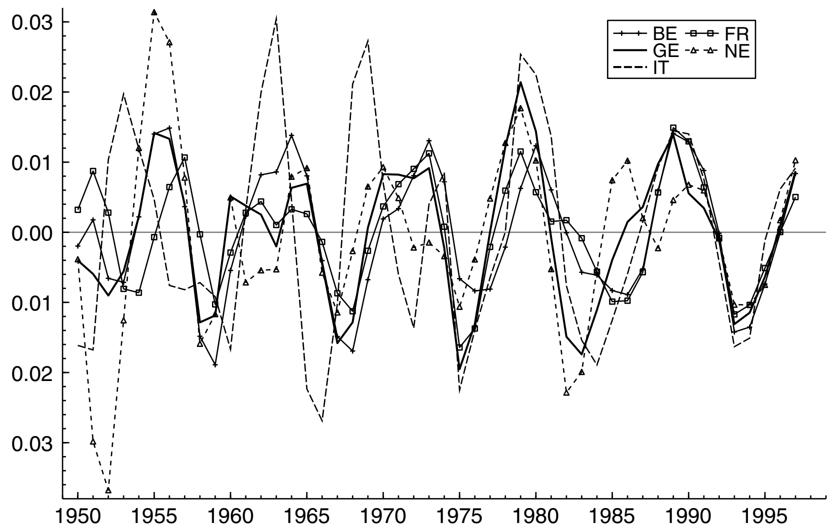

Figure 6. Cycles extracted from five core countries

Figure 6 shows the smoothed estimates of the cycles. While the dispersion at the beginning is quite large, the cycles are almost perfectly coordinated by the end of the 1990s. A plot of the standard deviation of the five cycles over time tells the same story.

As part of the convergence process, the cycle cross-correlations may well have changed over time so the figures have to be regarded as averages in some sense. Estimating the model over the period starting in 1970 gave much higher cross-correlations for the entries involving Italy and The Netherlands. The estimates of ρ and the period were equal to 0.90 and 5.90 years, respectively. As it stands, the cycle model does a good job of picking up the convergence, but if it were to be used to generate converging cycles, it would have to incorporate a mechanism that allowed the cross-correlations to tend gradually towards unity.

The same model was also fitted to the three poorer countries, Greece, Portugal and Spain. The damping factor and the period were found to be 0.81 and 7.29 years, respectively. The relatively low value of ρ indicates that the cycles are not particularly well-defined. This is especially true of Spain. Portugal's cycles are quite pronounced, with the variance being three times what it is in most countries. Although the cross-correlations are much lower than for the core group, a plot of the cycles shows that they are converging towards the end of the period, with a pattern not dissimilar to that in Figure 6.

6. TESTING REVISITED

As was noted earlier, standard unit root tests give a confused message with regard to convergence. This section examines some of the issues in testing for convergence in the light of the descriptive results of the last two sections. In doing so we draw on the recent investigation by Harvey and Bates (2003).

The first point to note is that the standard augmented Dickey–Fuller (ADF) test, with a constant included, has very low power. This can be illustrated by applying the test to the series on The Netherlands minus Italy. A graph shows very clear convergence. In 1950 the difference (in logs) between the two series was 0.534, corresponding to The Netherlands having a per capita income 1.7 times that of Italy, while in 1997 the gap was 0.105, corresponding to a ratio of 1.11. Yet the ADF test fails to reject. For example, with one lagged first difference,[4] the t-statistic is -2.14, as against a 10% critical value of -2.60. This is despite the fact that an initial value away from the mean actually helps to increase power. If the constant is dropped from the ADF regression, as is appropriate for a test of absolute convergence, then, with one lag, the t-statistic is -2.52, as against a 5% critical value of -1.96. Thus dropping the constant leads to a rejection of the null of no convergence at the 5% level of significance. The t-statistic is not quite significant at the 1% level, but if the information in all five core countries is pooled by forming a set of four contrasts with Italy, the multivariate homogeneous Dickey–Fuller t-test, proposed by Abuaf and Jorion (1990) and studied further in Harvey and Bates (2003), shows a very strong rejection.[5] Taking the observations from 1960 (since there is some divergence before 1960, although not between

[4] The result is relatively insensitive to the number of lags. For example, with three lags $t = -2.41$. (These additional lags are not statistically significant at the 10% level.)

[5] A LR test of the null hypothesis that the four contrasts are nonstationary is unable to reject at the 5% level of significance; whether or not intercepts are included makes no difference to the conclusion. Such a result is consistent with the simulations in Harvey and Bates (2003), which indicate that the LR test can have very low power relative to the multivariate homogeneous Dickey–Fuller t-test.

Table I. Trace LR statistic for cointegration with ($T - Np$) and without (T) degrees of freedom correction

R	T	$T - Np$	5% cv
0	70.57*	52.00	68.5
1	42.79	31.53	47.2
2	19.74	14.54	29.7
3	8.56	6.31	15.4
4	0.11	0.08	3.8

Note: * indicates a rejection at the 5% level of significance.

The Netherlands and Italy), the t-statistic, with one lag, is -3.04 while the 1% critical value (for $T = 100$) is -2.58. If all seven rich countries are considered, the rejection is even stronger as the t-statistic is -4.27. On the other hand, if the constant is included, even the multivariate tests are unable to reject at the 10% level.[6] Note that because the test takes account of cross-correlations between the series, it is invariant to the choice of benchmark.

The trace likelihood ratio test of Johansen (1988) may be used, as in Bernard and Durlauf (1995), to try to detect the number of cointegrating relationships and hence the number of clubs. However, the test is likely to have low power and hence to indicate too many clubs. For example, with the five core countries, again from 1960, the test, based on a model with an unrestricted constant[7] and two (p) lags, cannot reject the null hypothesis of five common trends, against the alternative that there are fewer, at the 5% level of significance when the degrees of freedom correction is used. In other words it indicates that there is no cointegration and no convergence clubs (with more than one member). With no degrees of freedom correction, four trends cannot be rejected. The results, obtained from the PcFIML program of Doornik and Hendry (2000), are shown in Table I; R is the number of cointegrating vectors so $5 - R$ is the number of (common) trends. The max test, not shown in the table, could not reject five trends even without the degrees of freedom correction.

7. CONCLUSION

Preliminary analysis from fitting a multivariate structural time series model to the 11 Euro-zone countries indicates two possible convergence clubs, one a high-income group, consisting of the five core economies plus Austria and Finland, and a low-income group, made up of Spain, Greece and Portugal. Ireland seems to follow its own growth path. The multivariate convergence model is successful in separating trends from cycles and capturing the absolute convergence in the two groups. The evidence for convergence in the second group is less compelling, but the assumption of a single common trend is not unreasonable. The groups themselves appear to have converged

[6] For five and seven countries the t-statistics were -2.26 and -2.25, respectively. The 10% critical values, from table III of Harvey and Bates (2003), are -3.73 and -4.26, respectively.

[7] This is appropriate (except possibly for $R = N - 1$) since the series are assumed to have drifts but no time trends in the cointegrating relationships. It could be argued that if clubs exhibit absolute convergence then constants should be excluded from the cointegrating relationships, but this is not a standard constraint.

in the relative sense. If this is correct, the implication is that the average per capita income in the poor group will remain almost 30% below that of the high-income group.

The cycles in the core high-income group show a remarkable coherence in recent years, with the group standard deviation having fallen dramatically. There is less coherence in the poor group, though again there is evidence of a movement towards the same cycle as for the rich group in recent years.

From the methodological point of view, the series illustrate the futility of trying to infer anything about convergence using univariate Dickey–Fuller tests with a constant included. However, it is possible to sensibly test against absolute convergence by dropping the constant and additional power is gained by pooling the information in several converging series. The likelihood ratio tests for cointegration appear to be of little use for detecting the number of convergence clubs.

Finally, the data available after 1997 makes it possible to examine recent aspects of convergence in the light of the modelling procedure. An artificial unified Germany series was constructed by splicing together the data for West Germany (up to 1997) and unified Germany (from 1992) by removing the average difference from the West Germany series. Trends were then fitted by STAMP and the common trend, constructed using the weights reported earlier, was subtracted. The main feature of a plot of these series is that the variability about the common trend has not changed very much. Germany continues its movement towards the common trend and there is a tendency for The Netherlands, and to a lesser extent Belgium, to become relatively richer. However, although the absolute convergence model predicts that countries above the common trend move towards it, the smooth convergence mechanism does allow for temporary divergence and the scale of the movements is not inconsistent with the estimated model. The cyclical components of the five core countries were also extracted. These have remained fairly close over the last four years, though not quite as close as for 1995–7.

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